7.A1 Which Boolean formulas are 3cnf-formulas?

(a) $(x_1 \wedge x_2 \wedge \overline{x_3}) \vee (\overline{x_1} \wedge x_2 \wedge x_3)$

(b) $(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2) \wedge (x_1 \vee x_3)$

(c) $(x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee x_3)$

7.A2 Given the 3cnf-formula below, construct the graph that is produced by the algorithm described in the proof of Theorem 7.32. For each 3-clique (if any) in the graph, determine the corresponding assignment of Boolean variables that satisfies the 3cnf-formula.

$$(x_1 \vee x_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_1} \vee \overline{x_2})$$

7.A3 Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and let $A \subset \Sigma^*$ be a language in NP with nondeterministic polynomial time decider N, with states $Q = \{q_0, q_1, q_{\text{accept}}, q_{\text{reject}}\}$, input alphabet Σ, and tape alphabet $\Gamma = \{\mathtt{a}, \mathtt{b}, \textvisiblespace, \times\}$. Suppose that N runs in time $O(n^3)$. Let w be the string `aaaba`.

(a) In the proof of Theorem 7.37, a tableau for N on w is described in general. Given the N and w above, what is the size of the tableau for N on w?

(b) The proof also describes a set, C, of symbols for the general tableau for N on w. Given the N and w above, explicitly list the elements of the set C.

(c) The proof also describes the variables $x_{i,j,s}$ for a Boolean formula ϕ that will simulate N on w. How many variables $x_{i,j,s}$ are there in the case of the N and w given above?

7.A4 Using the methods described in the proof of Corollary 7.42,

(a) Rewrite the Boolean formula $(x_1 \vee x_2) \wedge (\overline{x_1} \vee x_3)$ as an equivalent 3cnf-formla.

(b) Consider the Boolean formula $\phi = x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5$. Find a 3cnf-formula ϕ' (with the variables $x_1, \ldots, x_5$ as well as two new variables z_1 and z_2) with the property that ϕ is satisfied with some assignment $x_1 = a_1, \ldots, x_5 = a_5$ (where each a_i is 0 or 1) if and only if there is an assignment $z_1 = a_6$ and $z_2 = a_7$ such that ϕ' is satisfied with the assignment $x_1 = a_1,\ \ldots, x_5 = a_5,\ z_1 = a_6,\ z_2 = a_7$.

7.A5 Using the methods described in the proof of Theorem 7.46,

(a) Construct the directed graph G corresponding to the 3cnf-formula:

$$\phi = (x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2)\ .$$

(b) For each setting of the variables that satisfies ϕ, find every corresponding Hamiltonian path in the graph G.