Math 1031, Self-Evaluation Exercise 2: Solutions
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Name: (*Amy's Solutions*)

Discussion Section: *NA*

Discussion TA: *NA*

This exercise is for your practise. There are four open-ended problems. Give yourself 20 minutes to complete the exercise, and see how you do.

1. Consider $f(x) = 3^{1-x}$

   a.) What basic exponential curve can you use to help you graph this function?

   $$y = 3^{-x}$$

   b.) Graph the basic curve in (a).

   The curve is exponential *decay*. It goes through $(0, 1)$ (like every exponential.) And it goes through $(-1, 3)$ and $(1, \frac{1}{3})$.

   See hand-out for picture.

   c.) Graph $f(x)$.

   Rewrite $f(x)$ to make the transformation clearer:

   $$f(x) = 3^{-(x-1)}$$

   So we are transforming the basic curve $y = 3^{-x}$ by shifting to the *right* by 1.

   $$\begin{aligned} (0,1) &\longrightarrow (1,1) \\ (-1,3) &\longrightarrow (0,3) \\ (1,3) &\longrightarrow (2,3) \end{aligned}$$

   See hand-out for picture

2. Evaluate the expression:

$$\log_3\left(\frac{\sqrt[4]{27}}{3}\right)$$

We rewrite this using the properties of logarithms:

$$\begin{aligned}
\log_3\left(\frac{\sqrt[4]{27}}{3}\right) &= \log_3(\sqrt[4]{27}) - \log_3(3) \\
&= \tfrac{1}{4}\log_3(27) - \log_3(3) \\
&= \tfrac{1}{4}\log_3(3^3) - \log_3(3^1) \\
&= \tfrac{1}{4}\cdot 3 - 1 \\
&= \tfrac{3}{4} - 1 \\
&= -\tfrac{1}{4}
\end{aligned}$$

3. Solve the equation:

$$\ln(x+20) - \ln(x+2) = \ln x$$

First we need to combine the logarithms on the left hand side:

$$\begin{aligned}
\ln(x+20) - \ln(x+2) &= \ln x \\
\ln\left(\frac{x+20}{x+2}\right) &= \ln x
\end{aligned}$$

Then since logarithms are one-to-one, we can "drop the logs" to get an algebraic expression.

$$\frac{x+20}{x+2} = x$$

Now solve for x.

$$\begin{aligned}
\frac{x+20}{x+2} &= x \\
x+20 &= x(x+2) \\
x+20 &= x^2+2x \\
0 &= x^2+x-20 \\
0 &= (x-4)(x+5)
\end{aligned}$$

The algebraic equation has solutions $x = 4$ and $x = -5$. But we need to check these solutions to see if they make sense in the logarithmic equation. In particular, we need to check whether $x = 4$ and $x = -5$ are in the *domains* of the logarithmic functions in the logarithmic equation.

$\ln(4+20) = \ln(24)$	(fine)
$\ln(4+2) = \ln(6)$	(fine)
$\ln(4) = \ln(4)$	(fine)
$\ln(-5+20) = \ln(15)$	(fine)
$\ln(-5+2) = \ln(-3)$	(undefined!)
$\ln(-5) = \ln(-5)$	(undefined!)

So $x = 4$ is a solution to the original logarithmic equation, but $x = -5$ is not.

4. How long will it take \$1000 to be worth \$3500 if invested at 10.5% interest compounded quarterly?

The formula to use is:

$$A = P(1 + \tfrac{r}{n})^{nt}$$

Here $P = 1000$, and $r = .105$. We want to find t such that $A = 3500$, which means we have to solve the following equation for t:

$$\begin{aligned}
3500 &= 1000(1 + \tfrac{.105}{4})^{4t} \\
3.5 &= (1 + \tfrac{.105}{4})^{4t} \\
\ln(3.5) &= \ln\left((1 + \tfrac{.105}{4})^{4t}\right) \\
\ln(3.5) &= 4t \cdot \ln(1 + \tfrac{.105}{4}) \\
t &= \frac{\ln(3.5)}{4\ln(1 + \tfrac{.105}{4})} \\
t &\approx 12.09
\end{aligned}$$

It will take approximately 12.09 years for the investment to grow to \$3500.