Math 1031, Self-Evaluation Exercise 4
December 7, 2009

Name: (*Amy's Solutions*)

Discussion Section: *NA*

Discussion TA: *NA*

This exercise is for your practise. There are six open-ended problems. Give yourself 20 minutes to complete the exercise, and see how you do.

1. Dan, Yun, and Ben walk into McDonalds. Each picks one of these five options: hamburger, cheeseburger, chicken nuggets, grilled chicken sandwich, or a happy meal. What is the probability that at least one of them chooses a happy meal?

   Compute the probability of the complement i.e. the probability that none of them chooses a happy meal. Since three people each have four options (besides the happy meal) there $4 \cdot 4 \cdot 4 = 72$ ways they could choose their meals such that no one chooses a happy meal. Divide by the total number of ways they could choose their meals, $5 \cdot 5 \cdot 5 = 125$.

   $$P(E) = 1 - \mathrm{P}(E') = 1 - \frac{\mathrm{n}(E')}{\mathrm{n}(S)} = 1 - \frac{72}{125} \approx .49$$

   So the probability that at least one chooses a happy meal is about .49.

2. Out of 100 undergraduates, 15% are majoring in math, 10% are majoring in music, and 5% are double majoring in math and music.

   (a) What is the probability that a person from this group is majoring in math or music?

   If E is the event of majoring in math and F is the event of majoring in music, we are interested in computing $\mathrm{P(E \cup F)}$, the probability of a person majoring in math or music. We use the formula

   $$\mathrm{P(E \cup F) = P(E) + P(F) - P(E \cap F)}$$

   Well, $\mathrm{P(E)} = .15$, $\mathrm{P(F)} = .10$, and $\mathrm{P(E \cap F)}$ is the probability of majoring in both, so $\mathrm{P(E \cap F)} = .05$. So

   $$\mathrm{P(E \cup F)} = .15 + .10 - .05 = .2$$

   So the probability of a person majoring in math or music is 0.2.

   (b) What is the probability that a person from this group is majoring in math, given that he or she is majoring in music?

   The idea is that we look inside F. Use the formula,

   $$\mathrm{P(E|F)} = \frac{\mathrm{P(E \cap F)}}{\mathrm{P(F)}} = \frac{.05}{.10} = .50$$

   So the probability that someone is majoring in math, given that he or she is majoring in music is 0.5.

(c) Are the two events (majoring in music and majoring in math) dependent or independent?

Since the probability of a person majoring in math, given that he or she is majoring in music is *different* than the probability of a person majoring in math, that means that the events are dependent.

3. When you go home for break, your little brother challenges you to the following game. You are to draw a card at random from a deck. If you draw an ace he will pay you \$15, if you draw a face card (K,Q, or J), he will pay you \$10, but if you draw any other card, you will have to pay him \$5. Is this a fair game?

Compute your mathematical expectation of winning:

$$\begin{aligned} E_v \;=\; & (\text{probability of drawing an ace}) \times \$15 \\ & + (\text{probability of drawing a face card}) \times \$10 \\ & + (\text{probability of drawing any other card}) \times (-\$5) \end{aligned}$$

If this number is zero, it will be a fair game. Since there are four aces, the probability of drawing an ace is $4/52 = 1/13$, and since there are twelve face cards, the probability of drawing a face card is $12/52 = 3/13$. The probability of drawing any other card is $1 - 1/13 - 3/13 = 9/13$. So your mathematical expectation of winning is

$$E_v = \frac{1}{13} \times 15 + \frac{3}{13} \times 10 - \frac{9}{13} \times 5 = \frac{15}{13} + \frac{30}{13} - \frac{45}{13} = 0$$

So it is a fair game.

4. Each barista at Espresso Royale picks his or her favorite drink from these seven options: espresso, almond latte, hazelnut latte, hot chocolate, hot tea, smoothie, iced chai. What is the probability that Hannah picks one of the hot drinks, Rachel picks one of the lattes, and Genevieve picks an almond latte?

We can think of this as three independent events, one after the other, so the probability is

$$\text{P(a hot drink)} \times \text{P(any latte)} \times \text{P(pumpkin spice latte)} = \frac{5}{7} \times \frac{2}{7} \times \frac{1}{7} \approx .03$$

5. Seventy percent of patients report that a certain medical treatment is beneficial. If Fairview gave this treatment to 10 patients this year, what is the probability that exactly seven of those patients found the treatment beneficial?

Use the binomial probability experiment formula with $n = 10$, $x = 7$, $p = .70$.

$$\text{P}(x = 7) = \text{C}(10, 7) \cdot (.7)^7 \cdot (.3)^3 \approx .27$$