Math 170, Lab 5: Power Functions and Polynomials

Names: ______________________________

With your partner(s), read through the instructions and do the activities described. Only one report should be submitted from each group. This report is due Monday.

1. **Monomials**: Comparing graphs of $y = x^n$, where n is a whole number. Questions to think about : *What symmetries do the graphs have? Where are the functions increasing/decreasing? Are there local extrema? What is the end behavior? What points do the graphs have in common?*

   (a) Graph $y = x$, $y = x^3$, and $y = x^5$ on the same set of axes, using the `Plot` command. Start with an x-range of $[-2.35, 2.35]$ and a y-range of $[-1.5, 1.5]$ using the following command:

   ```
   Plot[{x, x^3, x^5}, {x, -2.35, 2.35}, PlotRange -> {-1.5, 1.5}]
   ```

   Also try a viewing rectangle of $[-5, 5] \times [-15, 15]$ and finally $[-20, 20] \times [-200, 200]$.

   Write a paragraph explaining how the graphs of $y = x$, $y = x^3$, and $y = x^5$ are alike and how they are different. Illustrate your points with sketches.

   (b) Now graph $y = x^2$, $y = x^4$, and $y = x^6$ on the same set of axes, with the following three viewing windows: first $[-1.5, 1.5] \times [-0.5, 1.5]$, then $[-5, 5] \times [-5, 25]$, and finally $[-15, 15] \times [-50, 400]$.

   Write a paragraph explaining how the graphs of $y = x^2$, $y = x^4$, and $y = x^6$ are alike and how they are different. Illustrate your points with sketches.

(c) Describe the differences between the graph of $y = x^n$ when n is even and n is odd. Illustrate your points with sketches.

(d) **Bonus**: Think about the inequality $x^n > x^m$. Is it always true when $n > m$? If not, for what x is it true? What about when $n < m$? Write a paragraph discussing under what conditions (on m, n, and x) the inequality is true. Illustrate your discussion with sketches.

2. **End Behavior/End Behavior Model** Remember that the end behavior of a function refers to the behavior of the function as x becomes very large (either positive or negative.) We use limit notation to denote end behavior. For example, when we write

$$\lim_{x\to\infty} f(x) \;=\; -\infty$$

we mean that, as x gets larger and larger in the positive direction, $f(x)$ gets larger and larger in the negative direction.

   (a) Sketch the graphs of x, x^2, x^3, x^4, x^5, and x^6 on the same set of axes, and identify the end behavior of each one.

   (b) Consider $f(x) = x^5 - 4x^4 - 9x^3 + 40x^2 - 4x - 4$ in relation to the six monomials above. Which monomial looks like this function for very large (positive and negative) values of x?

   (c) Such a function is called an "end behavior model" for $f(x)$. In *Mathematica*, graph the function $f(x)$ from (b) with the monomial that is the end behavior model on the same set of axes. Sketch your graph below.

3. **Power Functions**: Comparing graphs of power functions with either negative powers or fractional powers. Use the questions from (1.) to analyze the graphs.

   (a) Graph $y = x^{-1}$, x^{-2}, $y = x^{-3}$, and x^{-4} on the same set of axes, with the following three viewing windows: first $[0, 1] \times [0, 5]$, then $[0, 3] \times [0, 3]$, and finally $[-2, 2] \times [-2, 2]$.

   Write a paragraph explaining how the graphs are alike and how they are different. Illustrate your points with sketches.

   (b) Graph $y = x^{1/2}$, $x^{1/3}$, $y = x^{1/4}$, and $x^{1/5}$ on the same set of axes, with the following three viewing windows: first $[0, 1] \times [0, 1]$, then $[0, 3] \times [0, 2]$, and finally $[-3, 3] \times [-2, 2]$.

   Write a paragraph explaining how the graphs are alike and how they are different. Illustrate your points with sketches.

4. **Rational Powers**: Investigating the behavior of power functions of the form $f(x) = x^{m/n}$, where m and n are positive with no factors in common. Investigate each of the following cases:

   (a) n is even

   (b) n is odd and m is even

   (c) n is odd and m is odd

   For each case, use graphs to decide whether f is even, f is odd, or f is undefined for $x < 0$. Then, *confirm* your results *algebraically*. Write a paragraph explaining your observations, and illustrate your points with sketches.