Names: _____

With your partner(s), read through the instructions and do the activities described. See the *Mathematica* starter notebook for commands that you can edit. Only one report should be submitted from each group. This report is due Monday.

1. **Real and Complex Zeros of a Polynomial in Factored Form**: Consider the polynomial

$$p(x) = (x-2)^2(x-(3+i))(x-(3-i))$$

   (a) Without graphing $p(x)$, determine the zeros of $p(x)$. List the real and complex zeros of $p(x)$ with their multiplicities.

   (b) Without graphing, determine the x-intercepts of the graph of $p(x)$. For each x-intercept, state whether the graph actually crosses the x-axis at that point, or merely touches the x-axis.

   (c) Now use `Plot` to check your answer. Sketch the graph below, making the x-intercepts clear and labeling which ones cross the x-axes and which ones merely touch the x-axis.

(d) Use the command `Expand` to see the standard form of $p(x)$, and write it below.

(e) Let's see what would happen if I changed one of the complex zeros without changing the other. For example, consider

$$q(x) = (x-2)^2(x-(3+i))(x-(4-i))$$

Use `Expand` to see the standard form of $q(x)$, and write it below.

What do you notice?

2. **A Polynomial with Specified Real and Complex Zeros**: Suppose $r(x)$ is a polynomial with real coefficients of minimal degree that has a zero of multiplicity 2 at $x = -1$ and a zero of multiplicity 1 at $x = 3i - 4$.

(a) Write $r(x)$ in factored form.

(b) Find the standard form of $r(x)$ using `Expand`, and write it below.

(c) Without graphing, determine the x-intercepts of the graph of $r(x)$. For each x-intercept, state whether the graph actually crosses the x-axis at that point, or merely touches the x-axis.

(d) Now use `Plot` to check your answer. Sketch the graph below, making the x-intercepts clear and labeling which ones cross the x-axes and which ones merely touch the x-axis.

3. **Is it possible . . . ?** If so, include an example polynomial in standard form; if not, give a reason.

(a) Is it possible to find a polynomial of degree 3 with real number coefficients that has -2 as its only real zero?

(b) Is it possible to find a polynomial of degree 3 with real coefficients that has $2i$ as its only nonreal zero?

(c) Is it possible to find a polynomial of degree 4 with real coefficients that has zeros -3, $1+2i$, and $1-i$?

(d) Is it possible to find a polynomial of degree 4 with real coefficients that has zeros $1+3i$ and $1-i$?

4. **Modeling Circulation of Blood** Research conducted at a national health research project shows that the speed at which a blood cell travels in an artery depends on its distance from the center of the artery. The function $v = 1.19 - 1.87r^2$ models the velocity (in centimeters per second) of a cell that is r centimeters away from the center of an artery.

   (a) What is the domain of the modeling function, as a mathematical object?

   (b) What domain restrictions do the real-life situation impose on the model? (You may need to do some googling to find this.)

   (c) Use `Plot` to graph the model, on the domain that you specified in (b). Does the model make sense for all the values of r in this domain? Explain.

   (d) Tweak your viewing rectangle (using the `PlotRange` option to change the y-range) until you are satisfied that you have a meaningful viewing rectangle for this model. Sketch your graph below. Make sure to label your axes and include units.

   (e) If a blood cell is traveling at 0.975 cm/sec, estimate the distance the blood cell is from the center of the artery. Mark and label the relevant point on your sketch above.