Math 170, Lab 9: Exponential Functions

Names: ____

Only one report/worksheet should be submitted from each group. This report is due Monday.

1. **Comparison of power functions and exponential functions** In this problem, you will compare the graphs of power functions x^a and exponential functions a^x. For each pair below, sketch the two graphs on the same set of axes, identify the domain and range for each, and describe the end behavior of each, using limit notation.

   (a) Compare the graphs of $f(x) = x^2$ and $g(x) = 2^x$, as described above.

   (b) Compare the graphs of $f(x) = x^3$ and $g(x) = 3^x$.

   (c) Compare the graphs of $f(x) = x^{1/2}$ and $g(x) = (1/2)^x$.

2. **Solving equations involving exponentials** Solve the following equation graphically.

$$2^{x-1} = 3^x - 2$$

(Besides *stating* the solutions, you should *explain* how you found the solutions. Sketch a graph to support your explanation.)

3. **Carbon dating** Carbon 14 is radioactive and changes to nitrogen by radioactive decay. Cosmic rays create carbon 14 in the upper atmosphere at a rate which balances the decay. The fraction of carbon 14 to non-radioactive carbon in our atmosphere is constant. Since living plants take in CO2 from the atmosphere, the fraction of carbon 14 to non-radioactive carbon in living plants is the same as in the atmosphere. When a plant dies, the amount of carbon 14 in dead plant matter diminishes over time. The *percentage*, y, of carbon 14 compared to the atmospheric ratio in the plant x years after absorption is given by the following equation:

$$y = 10^{2-0.0000523x}$$

The half-life of carbon 14 is the time it takes to reach half, or 50%, of the original quantity.

(a) Use this equation to determine y when $x = 100$ years, 1000 years, and 10,000 years.

(b) Graph the equation and determine the half-life of carbon 14. (Sketch the graph below.)

(c) At an archaeological dig a piece of wood is found to have 2% of the atmospheric carbon 14 ratio. How old is the wood? (Solve the equation graphically, and make sure to include a sketch of the graph in your answer. You may round to the nearest 10,000 years.)

4. **Population growth** The population (in millions) of the United States for the years 1900 through 2000 is shown in the table below.

Year	Population (millions)
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7
2000	281.4

(a) Use *Mathematica* to generate a scatter plot of this data. Then, by guessing and graphing, find an exponential function of the form $f(x) = a \cdot b^x$ that fits the data as well as possible.

(b) Use *Mathematica* to find the exponential regression model for this data.

```
FindFit[YourData, a*b^x, {a,b}, x]
```

(c) Print out a graph that shows both of the two models (functions) above, together with the data points. (Make sure to include axes labels with units and labels telling which curve corresponds to which function.)

(d) It was announced in October of 2006 that the population of the U.S. had reached 300 million. How does this compare with the two models that you found above? What would they have predicted?

5. **Appendix: *Mathematica* commands** You might want to copy and paste some commands from the Lab 4 *Mathematica* notebook, but here's a brief refresher.

   (a) To enter data points, use a list of lists (curly braces), for example

   ```
   {{19, 22}, {21, 23}, {24, 25}}
   ```

   (b) To generate a scatter plot use the `ListPlot` command.

   (c) To superimpose the graph of a function onto a scatter plot, use the `Show` command, together with the `Plot` and `ListPlot` commands.

   (d) To add labels, use the `AxesLabel` option inside the `Plot` command.

   (e) To label curves on a graph, double click on the graph, then select "Drawing Tools" from the "Graphics" menu. You can add arrows, text, etc. to the graph.