Math 170, Lab 10: Modeling with Exponential, Logistic, and Logarithmic Functions

Names: ____

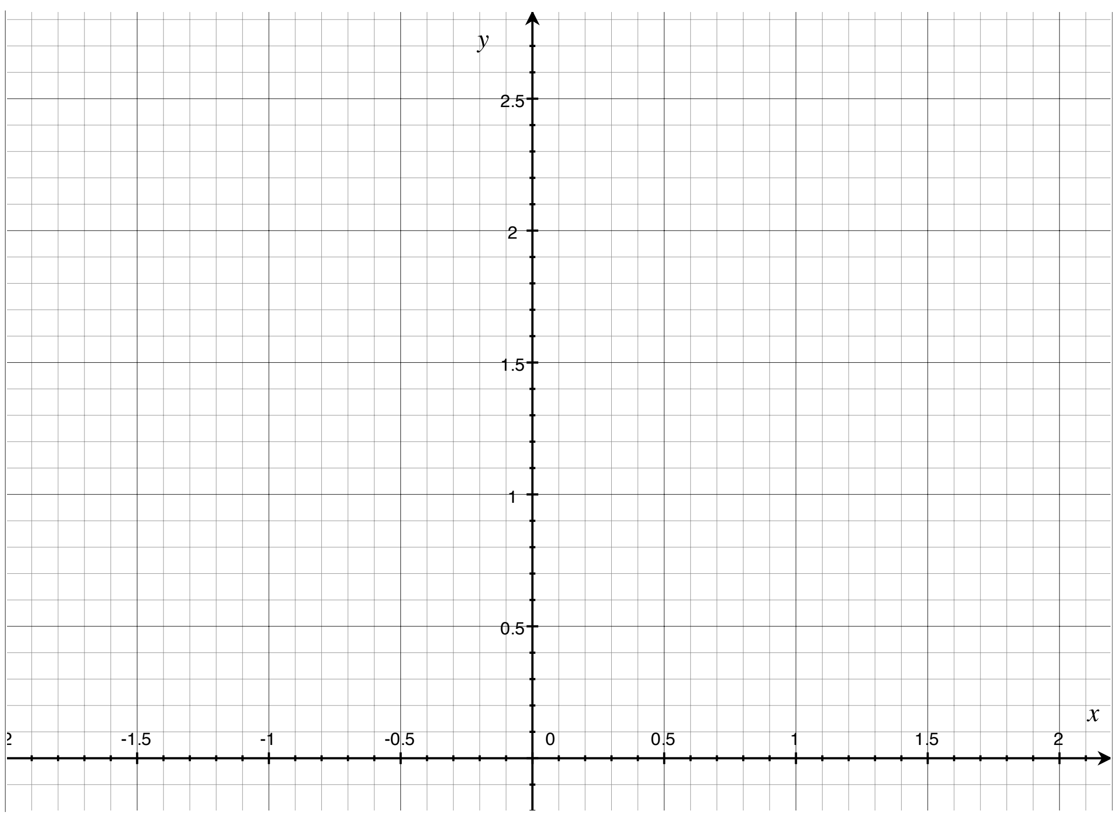
With your partner(s), read through the instructions and do the activities described. Only one report should be submitted from each group. This report is due Monday.

1. **Re-expressing data**: In class, we used *Mathematica* to obtain a power function model, $r = 233.2\, w^{-0.3167}$ for the pulse rate, r, in beats per minute, of a mammal as it depends on body weight w, in kilograms, using A. J. Clark's 1927 measurements. In this exercise, we will re-express the data, find a linear model by hand, and compare it to the power function model.

   (a) The table below shows A. J. Clark's measurements for r and w. Let $x = \log(r)$ and $y = \log(w)$. Fill in the table below with the x and y values (round to 3 significant figures.)

Mammal	w	r	$x = \log w$	$y = \log r$
Rat	0.2	420		
Ginea pig	0.3	300		
Rabbit	2	205		
Small dog	5	120		
Large dog	30	85		
Sheep	50	70		
Human	70	72		

   (b) Make a scatter plot of the (x, y) points obtained in (a) on the graph paper below.



(c) By hand, draw the line that seems to fit the data best. Calculate the slope and y-intercept of your line, and use this information to find a linear model for y as a function of x.

(d) Use your linear model to predict the pulse rate for a 450 kg horse. Is the result close to the 38 beats/min reported by A. J. Clark in 1927?

(e) State the difference, in the order of magnitude, between the body weight and the pulse rate of the horse and those of the rat.

(f) Find a power function model for r in terms of w, using your linear model $y = ax + b$ as a starting point. The goal is to get an equation of the form $r = C \cdot w^{\alpha}$, for some constants C and α. Since $y = ax + b$, we know that

$$\log r \;=\; a \cdot \log w \;+\; b$$

Rewriting this equation in exponential form and using the properties of logarithms and exponentials:

$$r \;=\; 10^{a \cdot \log w \,+\, b} \;=\; 10^{a \cdot \log w} \cdot 10^{b} \;=\; \left(10^{\log w}\right)^{a} \cdot 10^{b} \;=\; \ldots$$

2. The table below shows the populations of Alaska and Hawaii (in thousands) from 1900 to 2000. In this exercise, you will determine whether a linear, logarithmic, exponential, power, or logistic regression equation is the best model for the data for each state and explain your choice in paragraphs, with supporting tables and graphs as needed.

Year	Alaska	Hawaii
1900	63.6	154
1910	64.4	192
1920	55.0	256
1930	59.2	368
1940	72.5	423
1950	128.6	500
1960	226.2	633
1970	302.6	770
1980	401.9	965
1990	550.0	1108
2000	626.9	1212

   (a) Use *Mathematica*'s `FindFit` command to find linear, logarithmic, exponential, power, and logistic models for the population of Alaska in the 20th century. (See Regression Models on the course website for help with the *Mathematica* commands.)

   (b) Plot the models along with the data, on the same set of axes, and decide which model is the best.

   (c) Write a paragraph explaining your choice. Make sure to explain what you did (procedure), what you noticed (observations), and how you decided which model was best (conclusions). Include well-labeled graphs to support your writing. You may use a word processor or write your paragraph by hand.

   (d) Do the same for Hawaii.