**Math 170, Lab 14: Modeling with Trigonometric Functions II**

**Names:** ______________________________

With your partner(s), read through the instructions and do the activities described. Only one report should be submitted from each group. This report is due in class **Friday, April 13**.

1. Because its orbit is elliptical, the distance from the Moon to the Earth (measured from the center of the Moon to the center of the Earth) varies periodically. On Friday, January 23, 2009, the Moon was at its apogee (farthest from the Earth). The distance (in miles) of the Moon from the Earth each Friday from January 23 to March 27 is recorded in the table below.

| Date | Day | Distance |
|---|---|---|
| Jan 23 | 0 | 251,966 |
| Jan 30 | 7 | 238,344 |
| Feb 6 | 14 | 225,784 |
| Feb 13 | 21 | 240,385 |
| Feb 20 | 28 | 251,807 |
| Feb 27 | 35 | 236,315 |
| Mar 6 | 42 | 226,101 |
| Mar 13 | 49 | 242,390 |
| Mar 20 | 56 | 251,333 |
| Mar 27 | 63 | 234,347 |

(a) Draw a scatter plot of the data, with $x$ being the day and $y$ being the distance, in hundreds of thousands of miles.

(b) Approximately how far is the Moon from the Earth at apogee (farthest distance)?

Approximately how far is the Moon from the Earth at perigee (closest distance)?

Approximately how far is the Moon from the Earth on average?

What is the approximate number of days from apogee to apogee?

(c) Find a sinusoidal model $y = a\sin(b(x-h)) + k$ for the data by hand. (Again, let $y$ be the distance in hundreds of thousands of miles.) State the amplitude and period.

Check your answer with *Mathematica* by plotting the data and your sinusoidal model on one set of axes. Attach a print-out to this report.

(d) Since the data begin at apogee, perhaps a cosine curve would be a more appropriate model. Use the sine curve in part (c) and a cofunction identity to find a cosine curve that fits the data.

Again, check your answer with *Mathematica*, and attach a print-out.

2. **Damped Sinusoids** In this exercise, we explore the behavior of functions obtained by multiplying sine or cosine by another function.

   (a) Use *Mathematica* to graph $y = (\frac{x}{10})^2 + 1$ and $y = ((\frac{x}{10})^2 + 1)\cos(x)$ from $-4\pi$ to $4\pi$, on one set of axes. Sketch the graph below.

   Use *Mathematica* to graph $y = e^{-x/10}$ and $y = e^{-x/10}\sin(x)$ from $-4\pi$ to $4\pi$, on one set of axes, and sketch the graph below.

   In both cases, we still see wave-like behavior after multiplying sine or cosine by a function. What happens to the amplitude of the wave when we multiply by a function? What happens to the period?

   (b) A damped sinusoid can be used to model the motion of a pendulum or a spring subject to friction. Every-day experience dictates that a pendulum will not keep swinging forever, and an object suspended at the end of a spring will not bounce up and down forever. Multiplying a sinusoid by an exponential decay function is an effective way to model this behavior. Suppose that the damped sinusoidal model

   $$y = 0.22e^{-0.065t} \cdot \cos(2.4t)$$

   describes the displacement, $y$, in centimeters, of an object suspended at the end of a spring, from its equilibrium position as a function of time $t$, in seconds.

   What is the frequency of the oscillations?

   How long will it take before the oscillations are within 0.1 centimeter of the equilibrium position?