This is the second part of a two-part lab. Last week (in Lab 5) you began to explore the notion of the limit and its application to the definition of the derivative. You will write a report on Labs 5-6, which will be due 5pm Thurs Oct 13.

Goals of Lab 6:

1. To learn how to compute with numerical tables in *Mathematica*.
2. To understand how the notion of a *limit* underlies the definition of the definite integral.
3. To understand the definite integral of functions with certain symmetries.

The Lab Report for Labs 5-6

The lab report should be a thoughtful, well-written, and neatly organized document that summarizes both your experience in the lab and what you learned as a result of that experience. Keep in mind what you learned about good mathematical writing in Guide to Writing Mathematics, use the checklist from that document to evaluate your first draft, and revise your draft as needed before submitting your lab report. Each group of students will hand in a single report. Your report should contain the following parts:

1. Heading. At the top, list the title of the lab and the names of the people in your group.
2. Abstract. In one paragraph, summarize the purpose of this lab. State what it was that you were asked to explore.
3. Procedure and Observations. Summarize what you did in order to work through the exercises outlined for you, and present your results in a succinct, easy-to-grasp form, using tables or pictures or graphs with labels, where appropriate.
4. Conclusions. Write your conclusions in a paragraph or two. They should be inferences you draw from your data and calculations. Here is your opportunity to show that you understood the purpose of the lab, saw patterns in the data, and gained significant insights. Be as swooping in your conclusions as you dare, but back them up by explicit references to your data and calculations.

Submitting the Lab Report

Please submit the final draft of your lab report to the folder entitled "Lab5-6_Reports" in the Collaborative folder of the Math211 folder on the M-drive.

4. **The definite integral**: The definite integral

$$\int_a^b f(x)\,dx$$

gives the area under the curve $y = f(x)$ from $x = a$ to $x = b$. We can approximate the area under the curve using rectangles. To approximate the area under the curve using n approximating rectangles, we start by dividing the interval $[a, b]$ into n subintervals. Each subinterval will have length $\Delta x = (b - 1)/n$. The height of the approximating rectangle is given by the value of the function at either the left or the right endpoint of the subinterval. The area of the approximating rectangle is the height times the width. Adding up the areas of the approximating rectangles gives an approximation of the area under the curve. The definite integral is defined as the limit as n increases without bound:

$$\int_a^b f(x)\,dx \quad = \quad \lim_{n\to\infty} \sum_{i=0}^{n-1} f(a + i\Delta x)\,\Delta x \quad = \quad \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\,\Delta x$$

where $\Delta x = (b - a)/n$.

We will approximate the area under the curve for two functions using left and right approximating rectangles.

(a) Consider the function $f(x) = x^2 \cos x$. We will estimate the area under the curve from 0 to $\pi/2$. For our first approximation, we will use $n = 5$ approximating rectangles.

i. Begin by defining some things: Define $f(x)$ in *Mathematica* by typing

```
f[x_] = (x^2) * Cos[x];
```

Define the endpoints by typing:

```
a = 0; b = Pi/2;
```

Define the number of approximating rectangles by typing:

```
n = 5;
```

The width Δx of each approximating rectangle is the length of the interval $[a, b] = [0, \pi/2]$ divided by the number of approximating rectangles $n = 5$. Define the width by typing:

```
deltax = (b - a)/n;
```

Then the endpoints of the subintervals are $a + i\Delta x$ where i ranges from 0 to n. The y-values at those endpoints are $f(a + i\Delta x)$. These will be the heights of the approximating rectangles.

ii. Use the `Table` command to compute the heights of the left approximating rectangles. For the left heights, type

```
leftheights = Table[f[a + i * deltax], {i, 0, n - 1}]
```

The outputs of this command should be a list of y-values, corresponding to the heights of the approximating rectangles. To find the area of each rectangle, we multiply the height by the width. Since the width is always Δx, we can compute the areas simultaneously with the command

```
leftareas = deltax * leftheights
```

To add up all the areas use the `Total` command:

```
leftapprox = Total[leftareas]
```

This is the approximation of the area under the curve using left rectangles.

iii. To compute the approximation using right rectangles, define the right heights:

$$\texttt{rightheights} = \texttt{Table}[\texttt{f}[\texttt{a} + \texttt{i} * \texttt{deltax}], \{\texttt{i}, \texttt{1}, \texttt{n}\}]$$

Then, as before, multiply by Δx to compute the areas of the right rectangles and use the `Total` command to compute the sum of the areas of the right rectangles. This is the approximation of the area under the curve using the right rectangles.

iv. Approximate the area under the curve using left rectangles with $n = 10$ and $n = 100$ approximating rectangles. (Just change the command defining n and copy and paste the other commands.)

v. Use your approximations to estimate the definite integral

$$\int_0^{\pi/2} x^2 \cos(x)\, dx$$

(b) Estimate the definite integral

$$\int_0^{\pi/2} x^3 \cos(x)\, dx$$

using $n = 10$ approximating left rectangles.

5. **Symmetries**

(a) Graph the functions

$$f(x) = x^2 \cos x \quad \text{and} \quad g(x) = x^3 \cos x$$

from $-\pi/2$ to $\pi/2$. Describe the symmetries that you notice.

(b) Estimate the definite integrals

$$\int_{-\pi/2}^{\pi/2} x^2 \cos(x)\, dx \quad \text{and} \quad \int_{-\pi/2}^{\pi/2} x^3 \cos(x)\, dx$$

using $n = 20$ approximating left rectangles, and compare to your results in (4). What do you notice? Explain your results in light of the symmetries of $f(x)$ and $g(x)$.