

This lab is based on Project 1 (Harvesting and Logistic Growth) at the end of Chapter 10 of your textbook, which we began discussing in class on Wednesday. Please bring the hand-out from class to lab. You will work in groups of two or three and submit one lab report, which is due 5pm Monday December 5.

Groups for Lab 13:

Yohann Garner Luis	Nathaniel Shelby Grace	Gretchen Christian A. Becca	Joel Edgar Justin	Mitchell Robert Cristian G
Jacob G Jackson Garner	Aaron Alan Jordan	Zoe Jacob D. Kelly	Ben Minah	

Goals of Lab 13:

1. To learn how to generate slope fields using *Mathematica*.
2. To answer a question about sustainable harvesting by analyzing slope fields.
3. To write a clear, concise, compelling professional report.

The Story: Your group is a group of interns for an environmental engineer, Janice Bernstein, who works for a fishing company. Janice has asked you to research the sustainability of harvesting a certain population of fish. She has provided you with a differential equation modeling the dependence of the rate of population growth on the size of the population. She expects a report, containing your mathematical analysis and your policy recommendations, by Monday.

Written Report: Your report should be in the form of a *professional report*, with a working professional engineer as your intended audience. Use principles of good mathematical writing, as in described in Guide to Writing Mathematics. Each group of students will hand in a single report. Your report should contain the following parts:

1. **Heading.** At the top, include a title for your report and list the names of the people in your group.
2. **Abstract.** In one paragraph, summarize the purpose of your investigation, your principal findings, and your policy recommendation.
3. **Procedure and Observations.** Summarize what you did in order to work through the problems outlined for you, and present your results in a succinct, easy-to-grasp form, using tables or pictures or graphs with labels, where appropriate.
4. **Conclusions.** Write your mathematical conclusions in a paragraph or two, and explain the significance of your conclusions to sustainable harvesting. State your policy recommendations and make a case for them.

Submit the your report to the “Lab13_Reports” folder (in the Collaborative folder on the M-drive.)

Evaluation: I will evaluate your reports based on the quality of your writing and the correctness of the mathematics. Each individual in the group will need to fill out an evaluation form for your group (last page). I will take this into account when assigning individuals grades from the lab report.

Questions to Guide Your Investigation:

As discussed in class, when there is no fishing, the population of fish satisfies the differential equation

$$\frac{dN}{dt} = 2N - 0.01N^2$$

where N is the number of fish after t years, and when the fish are harvested at a rate of 75 fish per year, the population of fish satisfies

$$\frac{dP}{dt} = 2P - 0.01P^2 - 75$$

where P is the number of fish after t years. We now consider the effect of various levels of harvesting. When H is the rate of harvesting, in number of fish per year, the differential equation modeling the fish population is

$$\frac{dP}{dt} = 2P - 0.01P^2 - H$$

Rescaling (to make the slope fields look nicer), let z be the population of fish *in hundreds*. Then the differential equation for z is

$$\frac{dz}{dt} = 2z - z^2 - 0.01H$$

For $H = 75, 100, 200$:

1. Use *Mathematica* to obtain a graph of dz/dt against z . What are the intercepts?
2. Use *Mathematica* to generate a slope field for the differential equation for z .
3. What are the equilibrium values of z ? For what initial values of z does z increase? decrease?
4. Determine whether or not the equilibrium values of z are stable.

Now use your mathematical analysis of the differential equation for z to address the question about sustainable harvesting. For which level(s) of harvesting is there an initial condition such that the population of fish does not eventually die out?

There are many ways in which a group member can contribute to a group, including:

- asking good questions,
- explaining concepts,
- helping organize the group members' thoughts,
- recording observations,
- writing sections of the lab report,
- proofreading the lab report,
- etc.

Keeping this in mind, please describe each of your group members' contributions (including your own) by estimating the percentage of your group's total effort each person contributed.