The goals of this project are:

1. To practice good mathematical writing
2. To create supplementary graphics using *Mathematica*
3. To typeset mathematical expressions using *Mathematica*

Additional resources on writing well, formatting text in *Mathematica*, and typesetting with *Mathematica* are available on the course website.

In this lab, you will be writing up your solution to exercise 4.1.14, on Descartes' algebraic method for computing derivatives. Your solution should be in the form of a typed paper. You will use *Mathematica* for word processing, generating graphics, and mathematical typesetting. The Writing and Formatting tab and the Typesetting tab of the Classroom Assistant Palette or the Writing Assistant Palette can help you with formating and typesetting.

I have described steps to your process below in an outline form. Your paper should, however, be narrative. You should write in complete sentences that are grammatically correct, even if they include mathematical expressions.

This lab will be due on Jan 26.

1. Create a title for your paper, and start your document by stating the title and author (you.) Use a title cell for the title and a section cell for your name.
2. Write a brief introductory paragraph giving an overview of your paper. Describe the problem. Your description can be similar to the first four sentences in the statement of the problem, but use your own words.
   - Note how Shahriari uses mathematical symbols in his sentences. You can use an *inline* math cell to include mathematical expressions in the body of your text. Under the Math Cells portion of the Writing and Formatting tab, notice the Inline Math Cell option.
   - Notice how all Shahriari's sentences are grammatically correct, even if they include mathematical symbols.
3. Illustrate with a diagram.
   - To create a graphic, open a *new* notebook in *Mathematica* and use the `Plot` command to obtain a graph of $f(x) = \sqrt{x}$ and the circle that is tangent to the curve at $(16, 4)$. Then copy and paste the graph into your paper. Keep this "scratch" notebook open, so that you can use it for other computations and plots, as needed.
   - Note: in order to do this, you need to know the equation of the circle. However, not all of the work to get the equation of the circle is relevant to the solution of the problem, merely to the illustration. So the procedure and steps that you needed to go through to find the equation of the circle should *not necessarily* be included.
4. Now explain how the problem is solved.
   (a) You worked with an equation of a circle:

   $$(x - h)^2 + y^2 = r^2$$

   As always, make sure you define the variables you use. What is h? What is r? Be explicit and say that you are talking about a circle centered at $(h, 0)$ with radius r.

(b) This circle touches a point on the curve $y = \sqrt{x}$, so, at that point, the equation becomes

$$(x-h)^2 + (\sqrt{x})^2 = r^2$$

(c) This is now a quadratic equation in x:

$$x^2 + (1-2h)x + (h^2 - r^2) = 0$$

(d) A quadratic equation typically has two solutions, $x = x_1$ and $x = x_2$ and can be factored as

$$(x - x_1)(x - x_2) = 0$$

but our quadratic equation only has one solution, $x = 16$. So this quadratic equation can be factored as $(x-16)^2 = 0$. Comparing the two equations

$$x^2 - 32x + 16^2 = 0 \quad \text{and} \quad x^2 + (1-2h)x + (h^2 - r^2) = 0$$

allows us to determine values for h and r.

(e) We find the slope of the line connecting the center of the circle to the point of tangency. Here another diagram might be useful. To create the diagram you actually need to find the equation of the line. However, this work is not relevant to the solution of the problem, merely to the illustration. So the procedure and steps that you needed to go through to find the equation of the line should *not* be included.

(f) Once we know the slope of the radius of the circle through $(16, 4)$, we determine the slope of the line tangent to the curve $y = \sqrt{x}$ at $(16, 4)$.

(g) We can illustrate the solution by drawing the tangent line as well.

(h) We check our work using our known formulas and procedures of calculus. This can be part of your closing remarks.

5. Write a concluding paragraph. One point you could make in the conclusion is that Descartes' method allows one to find derivatives without appealing to the notion of limits. (That is why it is called an *algebraic* approach.) This notion of limits is troubling to some (infinity and infinitesimals are hard concepts to grasp) and has not yet been rigorously defined in this class. When you use known formulas and procedures of calculus, you are relying on this troublesome notion of limits, even if you are not aware of it.