Name: _____

In this lab you will work through exercises 3.3.13-3.3.17. You may work in groups of 2-3, but each student should fill out their own worksheet. No formal report is due for this exploration.

1. Recall Fermat's Little Theorem. State it below.

2. Let n be a positive integer. Consider integers m such that $1 \leq m \leq n$ and such than m and n have no common prime factors. The number of such integers m is denoted by $\phi(n)$.

   Find $\phi(n)$ for all integers $n = 1, 2, 3, \ldots, 14$. Fill in the table below. (Probelm 3.3.13).

n	1	2	3	4	5	6	7	8	9	10	11	12	13
$\Phi\ (n)$	1	1											

3. What is $\phi(p)$ when p is a prime? Make a conjecture. (Problem 3.3.13)

4. State Euler's Theorem. (Problem 3.3.14)

5. Suppose $m = p_1p_2$ where p_1 and p_2 are two distinct primes. What is $\phi(m)$? Explore and make a conjecture. (Problem 3.3.15)

p_1	p_2	p_1p_2	$\Phi(p_1p_2)$
2	3	6	2
2	5	10	
2	7	14	
3	5	...	
3	7		

6. Applications. (Problems 3.3.17-3.3.19) Use your conjecture in the previous problem to:

   (a) Find the remainder of 14^{81} when divided by 15.

   (b) Find the remainder of 10^{100} when divided by 21.

   (c) Find the remainder of 2^{1003} when divided by 33.