Name: _____

Instructions In this lab you will work through exercises 13.1.5 and 13.1.7. You may work in groups of 2-3, but each of you should turn in your own worksheet, which is due next Thursday in lab.

(13.1.5) Suppose $f(t)$ is a decreasing function and that we have a table of some of its values. Suppose that $g(x)$ is a function related to $f(t)$ by an integral. We will use two methods to approximate $g(1)$, namely approximating rectangles and local Taylor approximations. (See the statement of the problem in the book.)

(a) First find upper and lower bounds for $\int_0^1 f(t)\,dt$ using inscribed and circumscribed rectangles. (Include a table showing the known values of $f(x)$. Include a picture that illustrates the procedure.)

Use the bounds you found above to find upper and lower bounds for $g(1)$. (Include the definition of the function $g(x)$.)

(b) Find a reasonable linear approximation for $f(t)$ near $t = 0$.

We will approximate $g(x)$, using the linear approximation for $f(t)$. In the definition of $g(x)$, replace $f(t)$ with your linear approximation. Use this to approximate $g(1)$.

(c) Find an approximate Taylor polynomial of degree two for $g(x)$ near $x = 0$, and use this to approximate $g(1)$.

(13.1.7) In this problem we suppose that function $f(x)$ has a certain initial value and that we have a table with some values of its derivative. We want to approximate $f(2)$, and again we will use approximating rectangles as well as Taylor polynomials.

(a) Find a linear approximation for $f(x)$ at $x = 0$, and use it to approximate $f(1)$. (Include a table of known values of $f'(x)$. Include a picture that illustrates the procedure.)

Find a linear approximation for $f(x)$ at $x = 1$, and use it to approximate $f(2)$. (Include a picture illustrating the procedure. This picture should be a continuation of the previous picture.)

(b) Let $g(x) = \int_0^x f'(t)\,dt$. How are $f(x)$ and $g(x)$ related? (Write an equation.)

(c) Find upper and lower bounds for $f(2)$, which differ by less than $1/2$. (Include a picture.)

(d) For your conclusions in (c), what assumptions did you need to make about $f(x)$?