1. Notice that
$$Ax = \lambda x \quad \Leftrightarrow \quad 0 = \lambda x - Ax \quad \Leftrightarrow \quad 0 = (\lambda I - A)x,$$
where I denotes the identity matrix. Using this observation, what relationship is there between the matrix $\lambda I - A$ and the eigenvectors for A?

2. The collection of all eigenvectors corresponding to an eigenvalue λ for a matrix A (together with the zero vector) is called the *eigenspace* of λ. Using your answer from (1), show that $\lambda = 2$ is an eigenvalue of
$$A \ = \ \begin{pmatrix} 3 & 1 & 3 \\ 0 & 1 & -1 \\ 1 & 0 & 4 \end{pmatrix}$$
and find a basis for its eigenspace.

3. Recall that a 2×2 matrix is invertible if and only if its determinant is nonzero. What does the null space of the matrix $\big(\lambda I - A\big)$ tell you about whether or not $\big(\lambda I - A\big)$ is invertible?

4. Use (3) to figure out a way to find all the eigenvalues and corresponding eigenvectors for
$$A \ = \ \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}$$