1. Consider the matrix

$$S_{1,2} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $S_{1,2}A$ in terms of A.)

2. Consider the matrix

$$S_{2,3} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $S_{2,3}A$ in terms of A.)

3. Using those two examples as a starting point, think about what other matrices will have similar effects. Define $S_{3,4}$, $S_{1,3}$, $S_{1,4}$, $S_{2,3}$, and $S_{2,4}$, and describe the effects that they have on arbitrary 4×4 matrices.

4. Now consider $n \times n$ matrices. Define $S_{i,j}$ $(1 \leq i < j \leq n)$and describe the effect it will have on an arbitrary $n \times n$ matrix.

1. Consider the matrix

$$D_{1,5} = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $D_{1,5}A$ in terms of A.)

2. Consider the matrix

$$D_{2,5} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $D_{2,5}A$ in terms of A.)

3. Using those two examples as a starting point, think about what other matrices will have similar effects. Define $D_{3,5}$ and $E_{4,5}$, and describe the effects that they have on arbitrary 4×4 matrices.

4. Now consider $n \times n$ matrices. Define $D_{i,a}$ $(1 \leq i \leq n, a \in \mathbb{R})$and describe the effect it will have on an arbitrary $n \times n$ matrix.

1. Consider the matrix

$$T_{1,2,5} = \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $T_{1,2,5}A$ in terms of A.)

2. Consider the matrix

$$T_{1,3,5} = \begin{pmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $T_{1,3,5}A$ in terms of A.)

3. Consider the matrix

$$T_{2,3,5} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What effect does this matrix have on an arbitrary 4×4 matrix, A, when you multiply on the left? (Describe $T_{2,3,5}A$ in terms of A.)

4. Using those three examples as a starting point, think about what other matrices will have similar effects. Define $T_{1,3,5}$, $T_{1,4,5}$, $T_{2,4,5}$, and $T_{3,4,5}$ and describe the effects that they have on arbitrary 4×4 matrices.

5. Now consider $n \times n$ matrices. Define $T_{i,j,a}$ ($1 \leq i, j \leq n, a \in \mathbb{R}$)and describe the effect it will have on an arbitrary $n \times n$ matrix.