Consider the linear transformation of the plane T with the following matrix representation:

$$A = \begin{pmatrix} 0 & -4 \\ 2 & 5 \end{pmatrix}$$

1. First, you will factor A into the product of three matrices each representing a recognizable linear transformation of the plane:
   (a) Use the QR-factorization to write A as the product of an orthogonal matrix Q and an upper triangular matrix R.
   (b) Describe in words the linear transformation represented by the orthogonal matrix Q that you obtained in (a).
   (c) Write the upper triangular matrix R that you obtained in (a) as the product of a diagonal matrix, D, and an upper triangular matrix with 1s on the diagonal, N. Use the fact that
   $$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & ab \\ 0 & d \end{pmatrix}$$
   (d) Describe in words the linear transformations represented by the diagonal matrix, D, and the upper triangular matrix with 1s on the diagonal, N, that you obtained in (c).

2. Next, you will make a series of drawings showing the effect of the linear transformation T on the unit square. The linear transformation T is a composition of the three linear transformations represented by Q, D, and N. Remember that, to compute the effect of the linear transformation on a vector, you must procede from the inside out (not left to right). In this case, you first need to apply the linear transformation represented by N, then the one represented by D, then the one represented by Q.
   (a) Carefully draw the unit square.
   (b) In three steps (corresponding to the three linear transformations described in (1c)), carefully draw the transformation of the unit square by T. You don't need to use the matrices for this part; just use the verbal descriptions of the transformations. Make sure you have labeled the tick marks on your axes to show scale.
   (c) To check your work, compute $A(\hat{\imath})$, $A(\hat{\jmath})$, and $A(\hat{\imath} + \hat{\jmath})$, and draw these vectors on one set of axes. Does this picture correspond to the picture you drew in (b)?

3. Each group will turn in one write-up. This will be due in class next Wednesday, November 23. Make sure the names of all group members are at the top.
   (a) On a clean sheet of paper, write the factorization of A as the product of the matrices Q, D, and N that you found.
   (b) Carefully draw the effect of the linear transformation T on the unit square in three steps, as described in (2b), and explain each step by describing each of the three linear transformations in words.

4. Each group member will complete a group evaluation. This is also due in class next Wednesday, November 23.

Consider the linear transformation of the plane T with the following matrix representation:

$$B = \begin{pmatrix} 0 & -3 \\ 6 & 2 \end{pmatrix}$$

1. First, you will factor B into the product of three matrices each representing a recognizable linear transformation of the plane:

   (a) Use the QR-factorization to write B as the product of an orthogonal matrix Q and an upper triangular matrix R.

   (b) Describe in words the linear transformation represented by the orthogonal matrix Q that you obtained in (a).

   (c) Write the upper triangular matrix R that you obtained in (a) as the product of a diagonal matrix, D, and an upper triangular matrix with 1s on the diagonal, N. Use the fact that

   $$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & ab \\ 0 & d \end{pmatrix}$$

   (d) Describe in words the linear transformations represented by the diagonal matrix, D, and the upper triangular matrix with 1s on the diagonal, N, that you obtained in (c).

2. Next, you will make a series of drawings showing the effect of the linear transformation T on the unit square. The linear transformation T is a composition of the three linear transformations represented by Q, D, and N. Remember that, to compute the effect of the linear transformation on a vector, you must procede from the inside out (not left to right). In this case, you first need to apply the linear transformation represented by N, then the one represented by D, then the one represented by Q.

   (a) Carefully draw the unit square.

   (b) In three steps (corresponding to the three linear transformations described in (1c)), carefully draw the transformation of the unit square by T. You don't need to use the matrices for this part; just use the verbal descriptions of the transformations. Make sure you have labeled the tick marks on your axes to show scale.

   (c) To check your work, compute $B(\hat{\imath})$, $B(\hat{\jmath})$, and $B(\hat{\imath}+\hat{\jmath})$, and draw these vectors on one set of axes. Does this picture correspond to the picture you drew in (b)?

3. Each group will turn in one write-up. This will be due in class next Wednesday, November 23. Make sure the names of all group members are at the top.

   (a) On a clean sheet of paper, write the factorization of B as the product of the matrices Q, D, and N that you found.

   (b) Carefully draw the effect of the linear transformation T on the unit square in three steps, as described in (2b), and explain each step by describing each of the three linear transformations in words.

4. Each group member will complete a group evaluation. This is also due in class next Wednesday, November 23.

Consider the linear transformation of the plane T with the following matrix representation:

$$C = \begin{pmatrix} -4 & -2 \\ 0 & 2 \end{pmatrix}$$

1. First, you will factor C into the product of three matrices each representing a recognizable linear transformation of the plane:

   (a) Use the QR-factorization to write C as the product of an orthogonal matrix Q and an upper triangular matrix R.

   (b) Describe in words the linear transformation represented by the orthogonal matrix Q that you obtained in (a).

   (c) Write the upper triangular matrix R that you obtained in (a) as the product of a diagonal matrix, D, and an upper triangular matrix with 1s on the diagonal, N. Use the fact that

   $$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & ab \\ 0 & d \end{pmatrix}$$

   (d) Describe in words the linear transformations represented by the diagonal matrix, D, and the upper triangular matrix with 1s on the diagonal, N, that you obtained in (c).

2. Next, you will make a series of drawings showing the effect of the linear transformation T on the unit square. The linear transformation T is a composition of the three linear transformations represented by Q, D, and N. Remember that, to compute the effect of the linear transformation on a vector, you must procede from the inside out (not left to right). In this case, you first need to apply the linear transformation represented by N, then the one represented by D, then the one represented by Q.

   (a) Carefully draw the unit square.

   (b) In three steps (corresponding to the three linear transformations described in (1c)), carefully draw the transformation of the unit square by T. You don't need to use the matrices for this part; just use the verbal descriptions of the transformations. Make sure you have labeled the tick marks on your axes to show scale.

   (c) To check your work, compute $C(\hat{\imath})$, $C(\hat{\jmath})$, and $C(\hat{\imath} + \hat{\jmath})$, and draw these vectors on one set of axes. Does this picture correspond to the picture you drew in (b)?

3. Each group will turn in one write-up. This will be due in class next Wednesday, November 23. Make sure the names of all group members are at the top.

   (a) On a clean sheet of paper, write the factorization of C as the product of the matrices Q, D, and N that you found.

   (b) Carefully draw the effect of the linear transformation T on the unit square in three steps, as described in (2b), and explain each step by describing each of the three linear transformations in words.

4. Each group member will complete a group evaluation. This is also due in class next Wednesday, November 23.