Consider the symmetric matrix

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

1. Compute the eigenvalues of A, and, for each distinct eigenvalue λ, find a basis for the corresponding eigenspace V_λ.
2. For each pair λ and μ of distinct eigenvalues, compute the dot products of the basis vectors for V_λ with the basis vectors for V_μ. What do you notice?
3. Normalize each eigenvector to be a unit vector, and let Q be the matrix whose columns are the unit eigenvectors. Given your observation from (2), what kind of matrix is Q?
4. Given (3), how can you find Q^{-1} quickly (e.g. without using the Gauss-Jordan method)? What are the implications for diagonalizing A?