The transpose of a matrix A, denoted A^T, is the matrix obtained by switching the rows and columns of A. If A is given entry-wise by $(a_{i,j})$, then A^T is given entry-wise by $(a_{j,i})$.

1. Find the transpose of each of the following matrices:

$$A = \begin{pmatrix} 1 & -1 & 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$$

2. If a matrix A is of size $(n \times m)$, what size is A^T? What size is AA^T? What size is $A^T A$?

3. Let A and B be matrices, α a scalar, and n a positive integer. True or false:

   (a) $\left(A^T\right)^T = A$

   (b) $\left(A + B\right)^T = A^T + B^T$

   (c) $\left(\alpha A\right)^T = \alpha A^T$

   (d) $\left(AB\right)^T = A^T B^T$

   (e) $\left(A^n\right)^T = \left(A^T\right)^n$

4. Compute the sum of each of the following matrices with its transpose. What do you notice?

$$A = \begin{pmatrix} 1 & 3 & 9 \\ -3 & 5 & 2 \\ 0 & 4 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 7 & 11 & 2 \\ -5 & 3 & 1 \\ 2 & 9 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 12 & 4 \\ -3 & -2 & 5 \\ 8 & 2 & 1 \end{pmatrix}$$

5. For each of the matrices in the first exercise, compute the product of the matrix with its transpose and the product of the transpose with the matrix. What do you notice?