Let u and v be vectors in the plane. The following inequality is known as the "triangle inequality,"

$$\|u+v\| \;\leq\; \|u\| \;+\; \|v\|$$

1. Why is this name appropriate? Draw a picture which suggests that the inequality is true, and state what it means in plain English.

2. Use the geometric definition of the dot product to prove the following inequality:

$$|u \cdot v| \;\leq\; \|u\| \;\cdot\; \|v\|$$

   This is known as the "Cauchy-Schwarz" inequality.

3. Use the Cauchy-Schwarz inequality to prove the triangle inequality. (Hint: prove the corresponding inequality for the square of both sides.)