Math 109, Final Exam Study Guide Fall, 2014

The final exam will have 10 problems (with many parts) on 10 pages, and one page for scratch work. Make sure to show all your work and make your final answer clear. Include labels and units when appropriate. No notes, books, or calculators are permitted during the exam. The following formulas will be provided.

Sum and Difference Formulas

$$\cos(u+v) = \cos u \cos v - \sin u \sin v \qquad \cos(u-v) = \cos u \cos v + \sin u \sin v$$
$$\sin(u+v) = \sin u \cos v + \cos u \sin v \qquad \sin(u-v) = \sin u \cos v - \cos u \sin v$$

Double-angle Formulas

$$\sin(2u) = 2\sin u \cos u \qquad \cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

Power Reducing Formulas

$$\sin^2\left(\frac{u}{2}\right) = \frac{1-\cos u}{2} \qquad \cos^2\left(\frac{u}{2}\right) = \frac{1+\cos u}{2}$$

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1. (*20 points*) Fill in the blanks. Twenty blanks for basic formulas and identities and basic concepts (like tangent line and area). See previous exams and review sheets.

2. (*20 points*) Basic graphs: make sure you can recognize the graphs of the basic functions we have studied: exponential and logarithmic functions, trigonometric functions, inverse trigonometric functions. Make sure you understand basic graph transformations.

3. (*30 points*) Differentiation (without simplifying afterwards) as in Quiz 2 and Quiz 5. Additional problems involving derivatives from Chapter 11 are given at the end of the study guide.

4. (*30 points*) Evaluate the integrals, as in Quiz 2 and Exams 1 and 2. Additional problems that involve the antiderivatives we learned in Chapter 11 are given at the end of the study guide.

5. (*20 points*) Solving exponential, logarithmic, and trigonometric equations. See Exams 2 and 3.

6. (*10 points*) Implicit or logarithmic differentiation. See Exam 2. Additional examples using material from Chapter 11 are given at the end of the study guide.

7. (*20 points*) Application involving position and velocity. See Quiz 1 and Exam 1. Recall:
   (a) The derivative of a position function $s(t)$ is a velocity function $v(t)$, i.e. $s'(t) = v(t)$.
   (b) The position function $s(t)$ is an antiderivative of a velocity function $v(t)$. Initial conditions are required to find the position function, given the velocity function.
   (c) The net change in position (also called the displacement) from time $t = t_1$ to time $t = t_2$ is given by a definite integral: $\Delta s = s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t)dt$.

8. (*15 points*) Evaluating limits, using L'Hospital's Rule where appropriate. If using L'Hospital's Rule, make sure to identify the limit as an indeterminate form. Example problems are given at the end of the study guide.

9. (*15 points*) Absolute or relative extrema. See Exam 2. See also homework from 11.2 and 11.4.

10. (*20 points*) Area-so-far function and the Fundamental Theorem of Calculus. See Exam 1.

Practice Problems

Use the midterm exams, the midterm review sheets, the quizzes, and the homework for practice problems (in that order.) We have included some additional problems below for practice with material not covered on midterm exams. This material is integrated throughout the exam; approximately 35% of the exam relies critically on your understanding of Chapter 11 and Appendix B1.

1. Differentiate. (Do not simplify afterwards.)

   (a) $\frac{d}{dt}(\cos t + \sec t)$

   (b) $\frac{d}{dx}\ \tan(x)(\cot(x) + 1)$

   (c) $\frac{d}{d\theta}\ \sin(2\theta - \pi)$

   (d) $\frac{d}{dx}\ \sqrt[3]{\sin(x) + \pi}$

   (e) $\frac{d}{dt}\ \left(\arctan t\right)^7$

   (f) $\frac{d}{dx}\ \arcsin(5x)$

   (g) $\frac{d}{du}\ \ e^u \sin u$

   (h) $\frac{d}{dt}\ \ t^{4/3}\sec t$

(i) $\dfrac{d}{dx}\ \dfrac{\tan x}{\ln x}$

(j) $\dfrac{d}{dw}\ \dfrac{3w^2+10}{\cos(6w)}$

2. Evaluate the integrals.

   (a) $\int\ \sin t + \sin \pi\ dt$

   (b) $\int\ \cos(3\theta)\, d\theta$

   (c) $\int \sec^2\theta\ +\ \theta\ d\theta$

   (d) $\int \sec x(\sec x + \tan x)\, dx$

   (e) $\displaystyle\int \frac{3}{\sqrt{25-x^2}}\ dx$

   (f) $\displaystyle\int \frac{x^2+4}{x}\ dx$

(g) $\displaystyle\int \frac{3t}{t^2+1}\ dt$

(h) $\displaystyle\int \frac{1}{x^2+16}\ dx$

(i) $\displaystyle\int_0^{\frac{\pi}{8}} 3\cos(2x)\,dx$

(j) $\displaystyle\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \sin\theta\cos\theta\ d\theta$

3. (a) Given $y = (\cos x)^x$, find $\frac{dy}{dx}$ using logarithmic differentiation.
   (b) Given $y = \arctan(x)$, find $\frac{dy}{dx}$ using implicit differentiation.

4. Evaluate the limits, using L'Hospital's Rule where appropriate. If using L'Hospital's Rule, make sure to identify the limit as an indeterminate form.

(a) $\lim_{x \to \infty} \frac{\ln x}{x^2}$

(b) $\lim_{x \to 0} \frac{x}{\ln x}$

(c) $\lim_{x \to \infty} \frac{e^{2x}}{x^2}$

(d) $\lim_{x \to 0} \frac{x}{1 - \cos x}$

(e) $\lim_{x \to 0} x \csc x$