Math 109, Practice for Exam 3 November 17, 2014

Name: _____ **Section:** _____

Instructions: The exam will have 10 problems. Make sure to show all your work and make your final answer clear. Include labels and units when appropriate. No notes, books, or calculators are permitted during the exam. The following formulas will be provided on the exam.

Sum and Difference Formulas

$$\cos(u+v) = \cos u\cos v - \sin u\sin v \qquad \cos(u-v) = \cos u\cos v + \sin u\sin v$$
$$\sin(u+v) = \sin u\cos v + \cos u\sin v \qquad \sin(u-v) = \sin u\cos v - \cos u\sin v$$

Double-angle Formulas

$$\sin(2u) = 2\sin u\cos u \qquad \cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

Power Reducing Formulas

$$\sin^2\left(\frac{u}{2}\right) = \frac{1-\cos u}{2} \qquad \cos^2\left(\frac{u}{2}\right) = \frac{1+\cos u}{2}$$

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1. Fill in the blanks.

   (a) The period of $y = \tan x$ and $y = \cot x$ is _____ , whereas the period of the other four trigonometric functions is _____ .

   (b) The trigonometric functions _____ and _____ are even, but the remaining four trigonometric functions are odd.

   (c) The Pythagorean identity relating sine and cosine is: _____ .

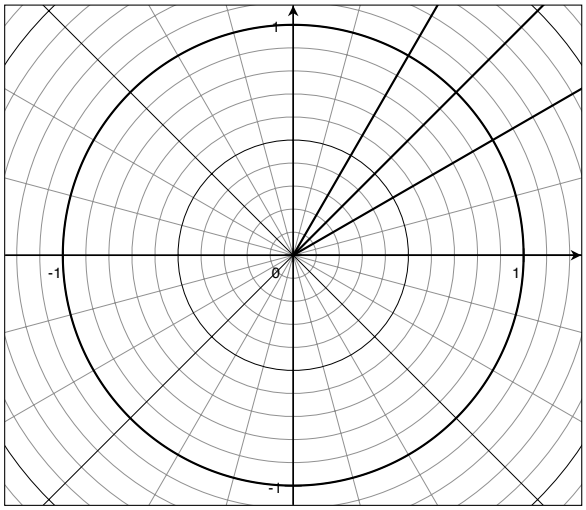
   (d) $\theta = \arccos(x) \iff x =$ _____ and _____ $\leq \theta \leq$ _____

   (e) The trigonometric function that is the reciprocal of $y = \cos x$ is $y =$ _____ .

   (f) For the simple harmonic motion described by $d = 3\cos(2t)$, the period is _____ , and the frequency is _____ .

   (g) The point $(x, 4)$ is 5 units away from the origin. If it lies on the terminal side of an angle θ in standard position, then $\sin\theta =$ _____ .

2. Use the axes provided to draw and label the points and angles described below.



(a) Label the five standard angles in the first quadrant in radians and degrees.

(b) Mark the points on the unit circle corresponding to the five standard angles in the first quadrant, and give x and y-coordinates for each point.

(c) Draw and label the following angles:

i. $\theta = \frac{7\pi}{6}$

ii. $\theta = -\frac{5\pi}{4}$

(d) Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for

i. $\theta = \frac{7\pi}{6}$

ii. $\theta = -\frac{5\pi}{4}$

3. Suppose that $\sin\theta = \frac{1}{4}$ and $\cot\theta < 0$. Find the values of the five remaining trigonometric functions by hand, and enter the values in the table below. Make sure to show all your work.

$\sin\theta$		$\csc\theta$	
$\cos\theta$		$\sec\theta$	
$\tan\theta$		$\cot\theta$	

4. Evaluate the expressions.

   (a) $\arccos(-\frac{1}{2})$

   (b) $\arctan(\sqrt{3})$

   (c) $\tan(\arcsin\frac{4}{5})$

5. Consider the function $f(x) = 4\sin(\frac{\pi}{3}x - \frac{\pi}{6})$

   (a) State the amplitude, period, and phase shift of $f(x)$.

   (b) Sketch a graph of $f(x)$. (Draw and label tick marks on both axes, to make the scale clear.)

6. State the domain, range, and period of each function, and then sketch the graph.

   (a) $y = \tan x$

   (b) $y = \sec x$

7. Perform the addition or subtraction and use trigonometric identities to simplify. Your final answer should be a single trigonometric function.

$$\frac{\tan x}{\csc x} + \frac{\sin x}{\tan x}$$

8. Verify the identities. (Make sure each step is clear and obvious.)

(a) $\dfrac{\cos x \csc x}{\cot^2 x} = \tan x$

(b) $\sin\left(\dfrac{\pi}{6} + x\right) = \dfrac{1}{2}\left(\cos x + \sqrt{3}\sin x\right)$

9. The sun is 30° above the horizon. Find the length of a shadow cast by a building that is 100 feet tall. (Leave your answer in exact form.)

10. Find the solutions of the following equations in the interval $[0, 2\pi)$.

    (a) $2\sin x - 1 = 0$

    (b) $4\sin x - 1 = 0$

    (c) $5\cos x - 2 = 0$

(d) $\sin 2x = \frac{\sqrt{3}}{2}$

(e) $2\cos^2 x = \cos x$

(f) $\sin 2x + 2\cos x = 0$