Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. the concept of the derivative
2. techniques for differentiation

1. The Concept of the Derivative

Rate of Change The derivative function tells the rate at which one quantity is changing with respect to another. For example, if $s = f(t)$ is the position of my car along Highway 13 t minutes after I turn onto the highway, then $f'(t)$ is the rate at which my position is changing with respect to time (i.e. the velocity of the car) t minutes after I turn onto the highway.

Slope Graphically, the rate of change can be interpreted as slope: if I graph the position s versus time t, the slope, at a given time value, t, is the velocity at time t.

Exercises.

1. Suppose that we have a function $f(x)$ that satisfies the following two conditions:

$$f(0) = 2 \qquad \text{and} \qquad f'(0) = -1$$

   (a) What is the slope of the line that is tangent to the graph of $f(x)$ at $x = 0$?

   (b) What is the equation of the tangent line?

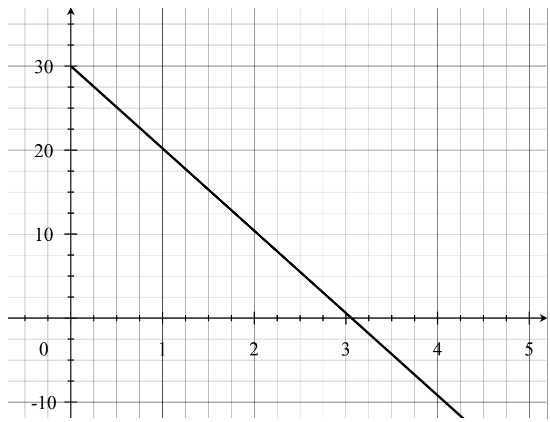
2. Suppose $g(2) = 5$ and $g'(2) = 3$. What is the tangent line to the graph of $g(x)$ at $x = 2$?

3. Suppose $h(x)$ is a function and $y = 4x + 5$ is the tangent line to the graph of $h(x)$ at $x = 1$.

   (a) What is $h'(1)$?

   (b) What is $h(1)$?

4. The graph below shows the velocity, $v(t)$, in meters per second, of a calculus book t seconds after it has been thrown in the air.



(a) After 1 second, is the calculus book moving up or down? After 4 seconds?

(b) At about what time is the calculus book momentarily paused?

(c) What does the graph of the derivative of $v(t)$ look like? What is the physical meaning of this? Explain in a complete sentence, and make sure to include units.

(d) **Challenge** Approximately how far did the calculus book move during the first second of its motion (from $t = 0$ to $t = 1$)? How about from $t = 1$ to $t = 2$? From $t = 2$ to $t = 3$?

(e) **Challenge** Assuming that the book was thrown from a height of 1 meter, sketch a rough graph of the position of the book during the first four seconds of its motion.

2. Differentiating Functions Using Formulas

The derivative of a constant function is always zero, because if a function is constant, it is not changing, so its rate of change is zero. For example:

$$\tfrac{d}{dx}5 = 0 \qquad \tfrac{d}{dt}14789 = 0 \qquad \tfrac{d}{dx}\sqrt{2} = 0$$

Power Rule Recall that, for any real number $a \neq 0$, the derivative of x^a is:

$$\tfrac{d}{dx}\, x^a \quad = \quad a \cdot x^{a-1} \qquad (a \neq 0)$$

For example, $\frac{d}{dx}x^3 = 3x^2$. This rule is especially useful when remember the following facts about powers:

$$x^{-a} \quad = \quad \frac{1}{x^a} \qquad \text{and} \qquad x^{1/n} \quad = \quad \sqrt[n]{x}$$

Combination Rules The combination rules are used to differentiate combinations (products, quotients, compositions) of functions:

- **Product Rule** $\frac{d}{dx}\big(u(x) \cdot v(x)\big) \; = \; u'(x) \cdot v(x) \; + \; u(x) \cdot v'(x)$
- **Quotient Rule** $\dfrac{d}{dx}\left(\dfrac{p(x)}{q(x)}\right) \; = \; \dfrac{p'(x) \cdot q(x) \; - \; p(x) \cdot q'(x)}{(p(x))^2}$
- **Chain Rule** $\frac{d}{dx}\big(F(u(x))\big) \; = \; F'(u(x)) \cdot u'(x)$

Note Before using one of the combination rules, it is always wise to see if it is possible to **rewrite** the function algebraically, to simplify differentiation.

Exercises

5. Differentiate.

   (a) $\frac{d}{dx}(3x + 5) \quad =$

   (b) $\frac{d}{dt}(t^5 + t^{-5}) \quad =$

   (c) $\frac{d}{dt}(\sqrt[3]{t} + \sqrt[3]{5}) \quad =$

   (d) $\dfrac{d}{dx}\;\dfrac{3}{x^4} \quad =$

6. Differentiate. Before using a combination rule, try to rewrite algebraically, if possible.

   (a) $\frac{d}{dx}\,\sqrt{x}(x+1) \quad =$

   (b) $\frac{d}{dx}\,x\sqrt{x+1} \quad =$

   (c) $\dfrac{d}{dx}\;\dfrac{\sqrt{x}+1}{x} \quad =$

   (d) $\dfrac{d}{dx}\;\dfrac{\sqrt{x}}{x+1} \quad =$

   (e) $\frac{d}{dx}\,\sqrt{x^2+1} \quad =$