Name: _____

Names of collaborators: _____

"[N]umbers have quirks of structure that endow them with personalities" (Strogatz, 9). We have seen, for example, that square numbers (like 4 and 9) can be represented by groups of rocks that can literally be arranged in a square pattern and that even numbers (like 2, 4, 6, ...) can be represented by groups of rocks that can be arranged in two even rows. In this activity, we will count the number of possible rectangular arrangements for a given number of rocks, and use this information to discover more "quirks of structure" of numbers.

Objectives. (1) Discover "quirks of structure" in numbers by working with them concretely, as "rock rectangles." (2) Practice the process of mathematical investigation. (3) Work collaboratively to investigate a mathematical question.

Instructions. We will start this activity together in class. You will work in your small groups to answer the questions below. Some questions are closed-ended (meaning that they have a specific answer); others are open-ended (meaning that there is room for many interesting and reasonable answers). If your group does not finish all the questions, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be one of two options for you to write about in your first report.

Basic Investigation

1. For a given number of rocks, there are several reasonable ways to count rectangular arrangements. As a group, you will need to agree on which arrangements "count" as rectangular arrangements and which do not. You will also need to agree on which rectangular arrangements count as "the same." Discuss this as a group, and clearly state below the rules for counting rectangles that your group agrees on.

2. For each number $1, 2, 3, \ldots, 12$, determine how many ways there are to arrange that many rocks into a rectangle. Record your work below, in a table with one column showing the number of rocks, the next column showing the rectangular arrangements for a given number of rocks (in words or pictures), and the third column showing the number of rectangular arrangements for a given number of rocks.

3. Observe and consider. What do you notice?

   (a) Here are some warm-up questions to get you started:

      i. Which number(s) of rocks can be arranged into the **least** number of rectangles?

      ii. Which number(s) of rocks can be arranged into the **greatest** number of rectangles?

      iii. Pick a number (of rocks) to look at. Write it here: _____ . Which **other numbers** (of rocks) can be arranged into the **same number of rectangles** as the number you picked?

      iv. Pick another number (of rocks) to look at. Write it here: _____ . Which **other numbers** (of rocks) can be arranged into the **same number of rectangles** as the number you picked?

   (b) Probe deeper; look for patterns and meaning.

   Remember that the goal is to use rock rectangles to discover "quirks" and "personalities" of different numbers: What does the number of rectangles tell us about the number of rocks?

   If you are having trouble, you could make a table that is the "reverse" of the first table. Make a column on the left for the number of rectangles, and a column on the right listing the number(s) of rocks that can be arranged into a given number of rectangles. Then try to see what the numbers of rocks that can be arranged into a given number of rectangles have in common.

Continued Investigation

4. Predict, conjecture and test.

   (a) Pick one pattern that you have observed, and state it in a full sentence as a **conjecture**. (A conjecture is a statement that you think is probably true, based on your investigation so far.)

   (b) Test your conjecture by looking at larger numbers (of rocks). (Go up to 16 or 20, or as high as you need to.) Does this test confirm your conjecture or contradict it in some way? If it contradicts your conjecture, you will need to reject or revise your conjecture.

Further Investigation

5. Continue looking for patterns, making and testing conjectures, until you have confirmed one or two more conjectures. (You may also try changing your rules for counting rectangles to see if any new "quirks" can be seen from this slightly different perspective. If you do this, be sure to make it clear what set of rules you are using when!)

6. Figure out and explain **why** at least one of your conjectures is true. (Once you have explained why a conjecture is true, it becomes a **conclusion**.)

Above and Beyond

7. Give explanations for as many of your conjectures as you can, and state your final conclusions.

8. As you bring your investigation to a close, what new questions do you think merit investigation?