Math 109, Activity #3: The Pythagorean Theorem and Squaring a Sum

Name: _____

Names of collaborators: _____

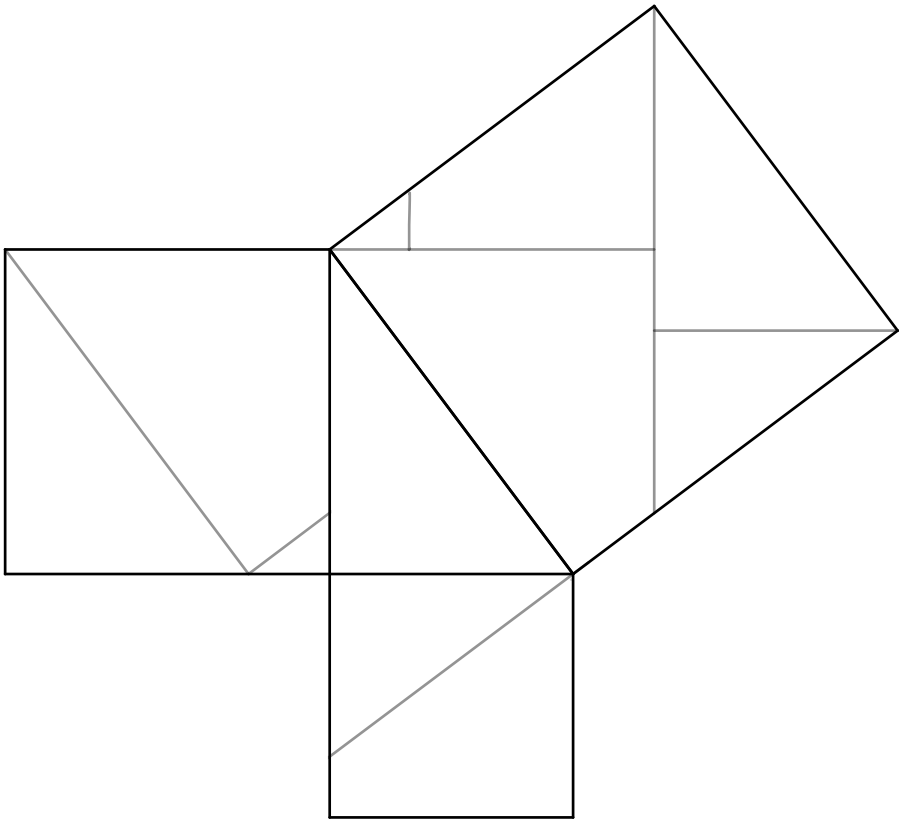
In this activity, we will illustrate the meaning of the Pythagorean theorem, and use illustrations to understand and explain other mathematical truths.

Objectives. (1) To understand the meaning of the Pythagorean theorem, using colors to illustrate a pictorial proof of the theorem. (2) To strengthen mathematical reasoning skills by developing proofs of algebraic and geometric facts using a combination of pictorial and algebraic arguments. (3) To improve collaborative problem-solving skills.

Instructions. We will start this activity together in class. You will work in your small groups to complete the activity outlined below. If your group does not finish, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be one of two options for you to write about in your weekly report.

Basic Investigation

1. Color the parts of the squares in the picture below to show that the combined area of the smaller squares is equal to the area of the largest square.



2. Explain in your own words what the Pythagorean Theorem says, and explain how your colored picture illustrates the truth of the theorem.

Continued Investigation

3. Use the picture below to explain why the formula $(a+b)^2 = a^2 + 2ab + b^2$ is true (at least for positive numbers a and b), following the outline below.

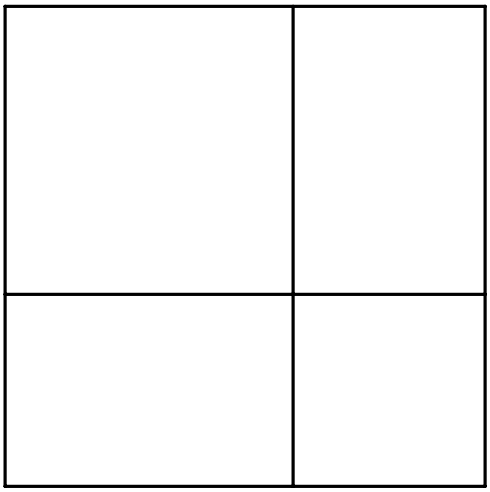
   (a) Find three squares in the picture. The largest one will represent $(a+b)^2$. The smaller ones will represent a^2 and b^2; you can decide which represents which.

   (b) Label the side lengths a and b in the picture, so that the smaller squares in the picture have the appropriate areas, a^2 and b^2. Color and label the squares of area a^2 and b^2.

   (c) Find and label the length $a+b$ in the picture. From your labeled picture, it should now be clear that $(a+b)^2$ is **not equal** to $a^2 + b^2$.

   (d) Find, color, and label two rectangles of area ab in the picture. When you combine these two rectangles with the smaller squares, what is the total area?

   (e) Explain in your own words how this proves that $(a+b)^2 = a^2 + 2ab + b^2$.



Further Investigation.

4. The picture below may be used to give a second explanation for why the Pythagorean Theorem is true. In this part of the investigation we will do some set-up work.

   (a) In this picture there are two squares, one larger (the outermost shape) and one smaller (inside the larger one). There are also four right triangles, each having exactly the same side lengths. Choose one right triangle to start with; label its legs a and b, and label its hypotenuse c.

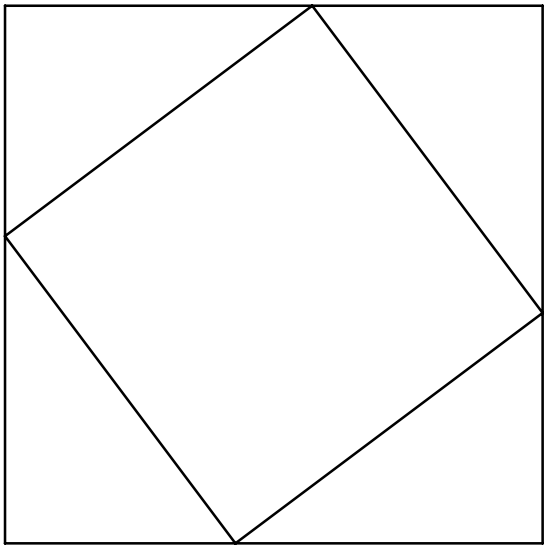
   (b) Now label the appropriate sides of the other three triangles. When you are done, each side of each triangle should be labeled a, b, or c. (Color-coding might be helpful too.)

   (c) What is the side length of the larger (outermost) square in the picture, in terms of a, b, and/or c? Use the side length to write down a formula for the area of the larger (outermost) square.

   (d) What is the side length of the smaller (inner) square, in terms of a, b, and/or c? Use the side length to write down a formula for the area of the smaller (inner) square).

   (e) What is the area of each of the four right triangles, in terms of a, b, and/or c?

   (f) Use addition to find the combined area of the four right triangles and the smaller (inner) square, in terms of a, b, and c.



Above and Beyond.

5. Use your work in the previous part to give a second proof of the Pythagorean Theorem, following the outline below.

   (a) In the previous part, you found two different formulas for the area of the larger (outermost) square. Set these two equal to each other to get an equation involving a, b, and c.

   (b) Starting with the equation you just wrote, use algebra to derive the relationship between a, b, and c that is the conclusion of the Pythagorean Theorem. Be careful to show every step.