Name: _____

Names of collaborators: _____

In this activity, we will first imagine that you have started a new business and are hiring your first employee. We will investigate how likely it is that a candidate is on drugs, given that he has a positive drug test. Then we will consider similar questions regarding a medical test for a rare disease called myasthenia gravis (MG).

Objectives. (1) To use a contingency table for a hypothetical population as a heuristic for calculating conditional probabilities. (2) To distinguish between similar-sounding conditional probabilities. (3) To appreciate the significance of distinguishing conditional probabilities in real life.

Instructions. We will start this activity together in class. You will work in your small groups to complete the activity outlined below. If your group does not finish, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be the activity to write about in your weekly report.

Basic Investigation

A certain drug test has a 98.6% sensitivity rate, meaning that 98.6% of drug users will test positive for drugs with this test. If your potential employee's drug test comes back positive, what does that mean? Does that mean there's a 98.6% chance he's taking drugs?

In order to analyze this this we need some additional information:

- The test has a 90% specificity rate, meaning that 90% of people *not* on drugs will test negative.

We also need to know the percentage of the general population who are on drugs.

1. It is estimated that 13.5% of the U.S. population (age 12 and above) uses drugs.

   (a) Consider a hypothetical population of 10,000 people. Fill out the contingency table to show the expected number of people in our hypothetical population in each category.

   Hint: Start by calculating how many people in our hypothetical population we expect to be using drugs. Then use the sensitivity rate to calculate how many of them we expect to test positive for drugs. Similarly, use the specificity rate to calculate how many of those not using drugs we expect to test negative for drugs.

	Using Drugs	No Drugs	Total
Pos. Test			
Neg. Test			
Total			

(b) What is the probability that a randomly selected person from the population, who has tested positive for drugs, is actually on drugs?

Continued Investigation

2. It is estimated that 39% of the U.S. population between the ages of 18 and 25 uses drugs.

   (a) Create a contingency table, as in the previous problem, to show what we expect for a hypothetical population of 10,000 18-to-25-year olds.

   (b) What is the probability that a randomly selected 18-to-25-year-old, who has tested positive for drugs, is actually using drugs?

3. We define events A, B, C, and D—each referring to people in the U.S.—as follows:

   - A, the event that a person, age 12 or above, uses drugs,
   - B, the event that a person, age 12 or above, tests positive for drug use,
   - C, the event that a person, age 18-25, uses drugs, and
   - D, the event that a person, age 18-25, tests positive for drug use.

   Look back at your work in the previous two problems to determine the following conditional probabilities:

   (a) $P(B \text{ given } A)$ _____

   (b) $P(A \text{ given } B)$ _____

   (c) $P(D \text{ given } C)$ _____

   (d) $P(C \text{ given } D)$ _____

Further Investigation

4. Myasthenia gravis (MG) is a rare disease affecting how the nerves communicate with muscles. Symptoms include weakness, fatigue, difficulty swallowing and difficulty breathing. It is estimated that about 17 out of every 100,000 people in the U.S. have MG. A blood test for MG has a sensitivity rate of 87.5% and a specificity rate of 97.5%.

   (a) Create a contingency table for a hypothetical population of 100,000 to show the expected numbers of people who have/do not have MG and who test positive/test negative for MG.

   (b) If a randomly selected person from the population tests positive for MG, what is the probability that this person has MG?

   (c) MG is more common among women younger than 40 and in men older than 60. A doctor suspects that a 75-year-old man with sudden muscle weakness and difficulty swallowing may have MG. Based on her prior experience, the doctor estimates that such a patient has a 30% chance of having MG, so she orders a blood test. The blood test comes back positive. Estimate the probability that the patient has MG.

   Hint: Create a contingency table for a hypothetical population of 1000 patients, who, like this man are in a higher risk group and show symptoms of the disease. Use the doctor's estimate that 30% of this group has MG.

Above and Beyond

5. For events E_1 and E_2, explain the difference between $P(E_1 \text{ given } E_2)$ and $P(E_2 \text{ given } E_1)$, and explain why being able to distinguish between these conditional probabilities is important in real life. Illustrate your point with examples.