Math 109, Activity #5: Completing the Square

Name: _____

Names of collaborators: _____

In this activity, we will use geometric reasoning ("completing the square") to solve quadratic equations.

Objectives. (1) To apply the method of "completing the square," to solve quadratic equations. (2) To appreciate the connections between geometry and algebra in the context of the quadratic formula. (3) To improve collaborative problem-solving skills.

Instructions. We will start this activity together in class. You will work in your small groups to complete the activity outlined below. If your group does not finish, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be one of two options for you to write about in your weekly report.

Basic Work

1. First we will solve $x^2 + 6x = 16$ using the method of completing the square. This means we must find a number x that makes the equation $x^2 + 6x = 16$ true.

   (a) Draw a small square and label its left and top sides "x". Then draw a long rectangle whose height is the same as the height of the square. Label the height "x" and the length 6.

   - What is the area of the square? _____
   - What is the area of the rectangle? _____
   - What is the combined area of the square the the rectangle? _____

   (b) Draw a picture showing how to break the long rectangle into two equally sized rectangles, each one still having a height of x.

   - What is the length of each of these two rectangles? _____
   - What is the area of each of these two rectangles? _____

(c) **On the left**, draw a picture showing the square and the two smaller rectangles arranged into a "notched" shape with your small square in the upper left corner. Label all the side lengths and areas. **On the right**, draw a square of area 16. **In the middle**, draw an equality sign to indicate that the areas of the shapes on the left and right are equal.

(d) Quickly copy your picture from the previous step below, so that you can add to it. Now "complete the square" by sketching a square in the negative space of the "notched" shape. Add a square of the same size to the right side as well.

- What is the area of the square you just added? _____
- What is the combined area of the two squares on the right side? _____

Now that the negative space of the "notched shape" is filled in, you should have a large square on the left (which is made up of two smaller squares and two rectangles).

- What is the side length of the large (outer) square on the left? _____
- What is the area of the large (outer) square on the left? _____

(e) Write an equation by setting the area of the large (outer) square on the left equal to the combined area of the two squares on the right. Solve this equation, i.e. find a number x that makes this equation true.

(f) Find a negative number x that also makes this equation true.

Continued Work

2. Solve $x^2 + 14x = 51$ by completing the square. Make sure to include well-labeled pictures illustrating the process.

Further Work

3. Suppose B and C are positive numbers. Solve $x^2 + (2B)x = C$ by completing the square.

Above and Beyond

4. The quadratic formula is a formula for the solutions of a quadratic equation. Given a quadratic equation of the form $ax^2 + bx + c = 0$, the two solutions are:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \text{and} \qquad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Use the quadratic formula to find the two solutions to the quadratic equations $x^2 + 6x = 16$ and $x^2 + 14x = 51$.

Hint: First you will need to get the equations in the proper form. For example, the first equation must be rewritten as: $x^2 + 6x - 16 = 0$. Then we can see that $a = 1$, $b = 6$, and $c = -16$.