Name: _____

Names of collaborators: _____

In this activity, we will estimate π, find a formula the area of a circle of radius r using the idea of a limit, and calculate distance and speed (the way the odometer and speedometer in a car do) using π.

Objectives. (1) To understand the definition of the number π though hands-on measurements, calculation, and applications. (2) To understand the notion of a limit, by applying it in a hands-on situation to derive a formula for the area of a circle, which involves π. (3) To improve collaborative problem-solving skills.

Instructions. We will start this activity together in class. You will work in your small groups to complete the activity outlined below. If your group does not finish, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be one of two options for you to write about in your weekly report.

Basic Work

1. **Estimating Pi.** The number π is defined as the ratio of the circumference of a circle to its diameter:

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

   (a) Neatly cut the edge off your paper paper plate so as to have a nice flat paper circle to work with.

   (b) Fold your circle in half and unfold it again. The crease will be a line going through the center of your circle. Measure the crease to find the diameter of your circle. (Include units.)

   Diameter: _____

   (c) Use a tape measure to measure the circumference of your circle. (Include units.)

   Circumference: _____

   (d) Convert any fractions in your measurements to decimals. For example, 7 and $\frac{3}{8}$ inches would be 7.375 inches, since $\frac{3}{8} = 0.375$.

   Diameter: _____

   Circumference: _____

   (e) Estimate π by calculating the ratio of your measurements for the circumference and diameter:

$$\pi = \frac{\text{Circumference}}{\text{Diameter}} \approx$$

We can get a formula for the circumference of a circle as follows. We start with the definition of π:

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

Then multiply both sides by the diameter:

$$\text{Circumference} = \pi \times \text{Diameter}$$

The radius is half the diameter so from here we get the formula:

$$\text{Circumference} = 2\pi \times \text{Radius}$$

Therefore the circumference of a circle of radius r is $2\pi r$.

2. **A Formula for the Area of a Circle**

Basic Work: (a)-(f)

(a) To color half the circumference of the circle, fold your circle in half along the crease you already made, and color the semicircular edge.

(b) The length of the semicircular edge is **half** of $2\pi r$. Use this fact to write down a formula for the length of the semicircular edge. (A formula, not a measurement.)

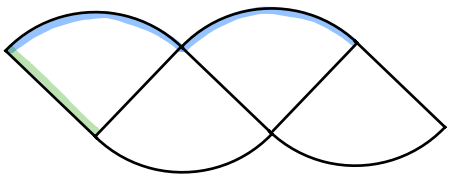
Length of semicircular edge: _____

(c) Unfold the circle and cut along the crease so that you have two half circles. Fold each in half, then cut along each crease, so that you have four quarter circles.

(d) Choose a new color, and color one of the straight edges of one of the quarter circles. How long is this edge? (The length is equal to a variable that we have already named; don't measure it.)

Length of straight edge of quarter circle: _____

(e) Arrange them to get a "scalloped rectangle," with both colored arcs on on top and the colored straight edge on the left, as shown below.



(f) Take a picture with your phone so that you can include it in your weekly report.

Continued Work: (g), (h), (j)-(m)

(g) Now fold each quarter circle in half, unfold, and then cut along the crease to get a total of eight pieces. Do this again to get a total of sixteen pieces.

(h) Arrange the pieces to get a "scalloped rectangle," with the colored arcs on on top and the colored straight edge on the left, and take a picture with your phone.

(i) If you have time, cut all the pieces in half again to get a total of 32 pieces, arrange them in a "scalloped rectangle," and take a picture. If you are running short on time, skip this step. (This will be counted as **Further Work**).

(j) What is the combined length of all the colored arcs on the top of your scalloped rectangle? (This should be a formula, not a measurement.)

Length of top edge of scalloped rectangle: _____

(k) What is the length of the colored edge on the left side of your scalloped rectangle?

Length of straight edge on left side of scalloped rectangle: _____

(l) If we pretend that the scalloped rectangle is a true rectangle, then its area would be the length of the top edge times the length of the side edge. Multiply these two quantities to get the area of the scalloped rectangle.

Area of scalloped rectangle: _____

(m) How is the area of the scalloped rectangle related to the area of the circle? What can you conclude about the area of a circle of radius r?

Above and Beyond

3. **Distance and Speed.** The odometer and speedometer in a car calculate distance and speed by counting the number or rate of wheel revolutions. Note that one revolution of the wheel corresponds to traveling a distance equal to the circumference of the wheel. So the number π is important in the calculations done by the odometer and speedometer.

   Here are some facts that are helpful for answering the questions below:

   - The tires on my Corolla have a diameter of 25.1 inches.
   - There are 12 inches in a foot and 5280 feet in a mile.
   - There are 60 minutes in an hour and 60 seconds in a minute.

   (a) The odometer of my Corolla measures 3,216 wheel revolutions in my drive from home to work. How many miles is it from home to work?

   (b) The speedometer of my Corolla measures my wheels rotating at a rate of 6.7 revolutions per second. How fast am I driving, in miles per hour?