Name: _____

Names of collaborators: _____

In this activity, we will explore the symmetries of an equilateral triangle.

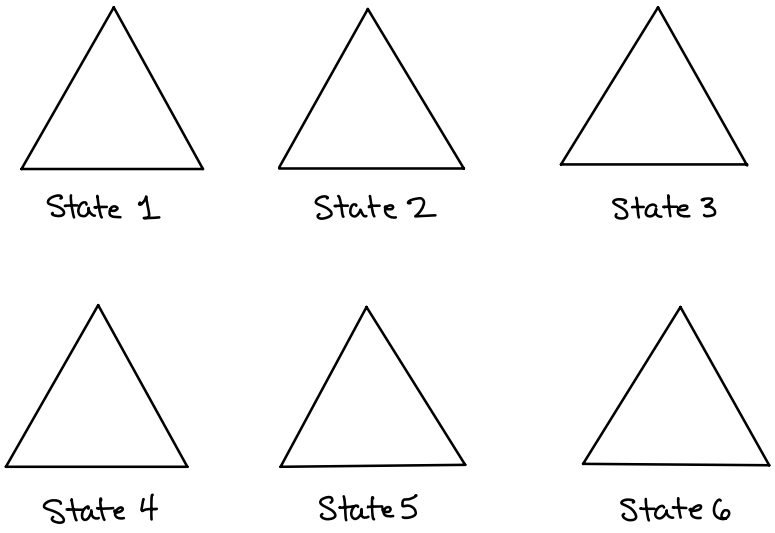
Objectives. (1) To get a taste of modern algebra by studying symmetries of an equilateral triangle as transformations. (2) To strengthen mathematical investigative skills. (3) To improve collaborative problem-solving skills.

Instructions. We will start this activity together in class. You will work in your small groups to complete the activity outlined below. If your group does not finish, you may continue working on the activity outside of class, either with your group or individually. The work you write on your paper should reflect your own understanding of the material. This activity will be one of three options for you to write about in your weekly report.

Basic Work

1. Place your equilateral triangle in its "frame." Draw it below, including the labels on the corners. (You may also find it helpful to color-code the corners.) This positioning will be considered your original or first "state."

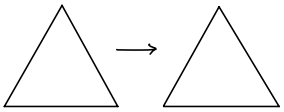
2. **States** Take your triangle out of its frame, and find all ways to place the equilateral triangle back in its frame. (You should end up with six "states," including the original one.) Draw each state below.



3. **Transformations** In words and with pictures, describe all the transformations of the equilateral triangle. The first transformation is the "do nothing transformation." There are five other transformations; each one takes the first state to one of the other six states.

   (a) To get started, put your triangle back in its original position. How can you get from this first state to the second state in one move?

   - Your answer will either be a rotation or a flip. Which one is it?
   - If it's a rotation, are you rotating clockwise or counter-clockwise? How far? (Rotating all the way around would be 360°, half of that would be 180°, a third would be 120°, etc..)
   - If it's a flip, what line are you flipping across?



   (b) Put your triangle back in its original position. How can you get from this first state to the third state in one move? (Describe in words and with pictures.)

Continued Work

   (c) Put your triangle back in its original position. How can you get from this first state to the fourth state in one move? (Use words and pictures.)

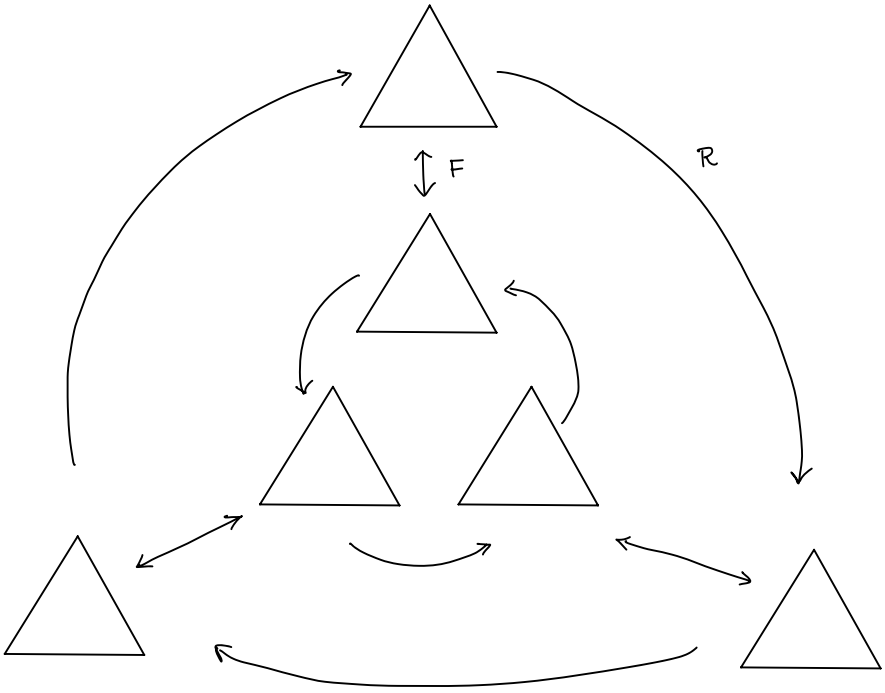
   (d) Put your triangle back in its original position. How can you get from this first state to the fifth state in one move? (Use words and pictures.)

   (e) Put your triangle back in its original position. How can you get from this first state to the sixth state in one move? (Use words and pictures.)

In the end, you should have: the "do nothing" transformation, two rotations, three flips.

Further Work

4. **Minimalist Approach.** Even though we have five "do something" transformations, it is possible to get from the first state to all other states using combinations of just **two** out of those five transformations. Make a diagram showing how to get from the first state to each other state using combinations of one rotation (call it R) and one flip (call it F).



Above and Beyond

5. **Commutativity.** Does this transformation group obey the commutative law? (No.) Give an example to show that not all transformations commute. (When we say that two transformations commute, that means if we start in a particular state, then do one transformation after the other, we will always end up in the same state, no matter which transformation we did first and which we did second.)