Math 109, Theoretical Probabilities Worksheet

Name: _____

Names of collaborators: _____

In this worksheet, we consider an experiment where we flip three (fair) coins in a row. This is one option for your next weekly report.

Basic Work

1. Write out all the possible outcomes of this experiment.
   - For example, write HHH for the outcome of flipping three heads in a row and HHT for the outcome of flipping two heads then a tail. (There will be 8 outcomes total.)

2. For each of the events below, (i) write out the outcomes corresponding to the event; then (ii) calculate the probability of the event using the formula:

$$P(E) = \frac{\text{number of outcomes corresponding to event } E}{\text{total number of equally-likely outcomes}}.$$

   (a) A, the event that we flip **all heads**

   (i)

   (ii)

   (b) B, the event that we flip **at least one tail**

   (i)

   (ii)

(c) C, the event that we flip **at least two heads**

(i)

(ii)

(d) D, the event that we flip **heads for the first coin**

(i)

(ii)

(e) F, the event that we flip **heads for the second and third coins**

(i)

(ii)

(f) D or F

(i)

(ii)

Continued Work

3. In this problem, we refer to the events A, B, C, D, and F from the previous problem.

   (a) Which event, B, C, D, or F, is the **complement**, A^c, of A? Looking back at the relevant part of the previous problem, check that the formula $P(A^c) = 1 - P(A)$ holds true.

   (b) Which event, A, B, or C is the same as the compound event D and F? _____

(c) Calculate $P(D \text{ or } F)$ using the formula below, and compare to your answer in 2f.

$$P(D \text{ or } F) \;=\; P(D) \;+\; P(F) \;-\; P(D \text{ and } F)\,.$$

Further Work

4. In this problem, we continue to reference the events A, B, C, D, and F from above. Calculate the following conditional probabilities, in two ways:

   (i) by counting outcomes:

   $$P(E_1 \text{ given } E_2) \;=\; \frac{\text{number of outcomes corresponding to the event } (E_1 \text{ and } E_2)}{\text{number of outcomes corresponding to the event } E_2}\;,$$

   (ii) and using the formula:

   $$P(E_1 \text{ given } E_2) \;=\; \frac{P(E_1 \text{ and } E_2)}{P(E_2)}\;.$$

   (a) $P(D \text{ given } C)$

   (i)

   (ii)

   (b) $P(D \text{ given } F)$

   (i)

   (ii)

(c) $P(A \text{ given } D)$

(i)

(ii)

(d) $P(A \text{ given } F)$

(i)

(ii)

Above and Beyond

5. For each conditional probability that you computed in the previous problem, (a) determine whether the two events are independent or dependent. In each case, (b) explain in your own words why it makes sense intuitively that the events are independent (or dependent). (*You will probably need additional space to answer this question; you may attach your extra work with a staple to this packet.*)