Name: ______________________________

Names of collaborators: ______________________________

Main Points to Review:

1. Functions, models, graphs, tables, domain and range
2. Algebraic functions, power functions, exponential functions, trig functions

1. Functions

A function is a relationship between two quantities, in which one quantity (e.g. position within this room) uniquely determines the other (e.g. air temperature). Given a position in the room, the air temperature is uniquely determined. Notice that the inverse relationship *cannot* be modeled with a function: we cannot consider position as a function of air temperature, because there may be many positions in the room that have the same air temperature. Said differently, the air temperature does not uniquely determine a position in the room.

There are (at least) four ways to represent a function: verbal (words), numerical (table), graphical, and symbolic (formula).

Exercise:

1. Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

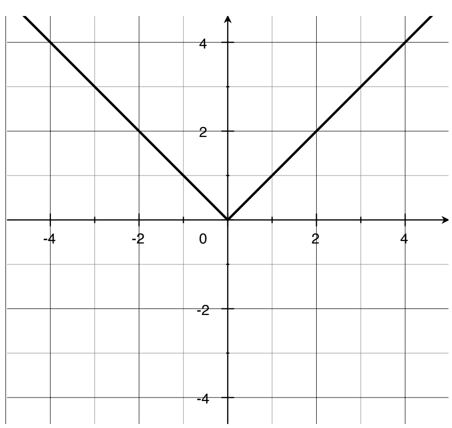

Figure 1: Is this function of x? Y/N

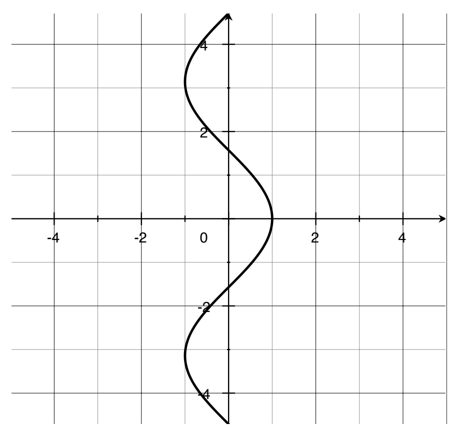

Figure 2: Is this function of x? Y/N

2. Algebraic Functions and Power Functions

Linear functions, and polynomial functions more generally, are very familiar and well-behaved. In particular, the domain of a polynomial is all real numbers. Rational functions are ratios of polynomials. These are relatively well-behaved but may have domain restrictions and/or asymptotes. In particular, if the denominator is zero for a specific x-value, that x-value is excluded from the domain.

Power functions are of the form $f(x) = x^a$, where a is a nonzero constant. There is quite a variety of functions in this family, including:

- familiar monomials: x^2, x^3, x^4 etc.
- root functions: $x^{1/2} = \sqrt{x}$, $\quad x^{1/3} = \sqrt[3]{x}$, $\quad x^{1/4} = \sqrt[4]{x}$, etc.
- the reciprocal function: $x^{-1} = \frac{1}{x}$

Note: Some of these functions have domain restrictions and/or asymptotes!

We use the general term "algebraic function" to refer to any function that is a combination of polynomial, rational, and root functions.

Exercise:

2. Classify each function given below, by circling the appropriate name or names.

   (a) $f(x) \;=\; 2x - 1$

   root power linear polynomial

   (b) $g(x) \;=\; x^3$

   algebraic power root polynomial

   (c) $h(x) \;=\; \dfrac{3x}{x+1}$

   algebraic power rational root

   (d) $Q(t) \;=\; t^{1/8}$

   power root exponential algebraic

   (e) $P(x) \;=\; x^5 - 3x - 1$

   linear power algebraic polynomial

3. Find the domain of the functions. Write your answer in interval notation.

   (a) $f(x) = 2/(3x - 1)$
   (b) $g(x) = \sqrt{10 - x}$.

3. Exponential Functions

Exponential functions come up frequently in modeling real world phenomena. Examples include population growth and radioactive decay. Exponential functions are characterized in the following way: given a certain unit of time (say one day) the amount of something (say the number of bacteria in a petrie dish) will increase (or decrease) by the same factor or ratio. So in the case of bacteria, we might say that the population doubles each day. This is exponential growth. If we start with 100 bacteria, the amount of bacteria would be modeled by the following equation:

$$P(t) \quad = \quad 100 \cdot 2^t$$

where $P(t)$ is the number of bacteria after t days.

Exponential decay is the reverse: the amount of a certain radioactive substance decreases by the same ratio every year, say by one-half. Then if we start with 100 grams, the next year there will be 50 grams, then 25 grams, etc. Here the amount of radioactive substance would be modeled by

$$A(t) \quad = \quad 100 \cdot (1/2)^t \quad = \quad 100 \cdot 2^{-t}$$

where $A(t)$ is the amount of radioactive substance left after t years.

There are a few things to notice about exponential functions $f(x) = a^x$ from their graphs:

- They have only positive outputs.
- The line $y = 0$ is a horizontal asymptote.
- If $a > 1$, the function is increasing constantly; if $0 < a < 1$ it is decreasing constantly.

Laws of exponents: for $a > 0$ and any real numbers x and y,

- $a^{x+y} = a^x a^y$
- $a^{x-y} = \frac{a^x}{a^y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

Exercise

4. A bacterial culture starts with 500 bacteria and triples in size every hour.
   (a) Find a function to express how many bacteria there are after t hours.
   (b) Sketch a graph of the function in (a). Make sure to include any asymptotes and intercepts.

5. A radioactive substance decays to one-third of its previous amount each year.
   (a) Find a formula to express how many grams of the substance there are after t years, if the initial size of the sample is 200 grams.
   (b) Sketch a graph of the function in (a). Make sure to include any asymptotes and intercepts.

6. Use the laws of exponents to rewrite and simplify the expressions:
   (a) $x(2x^2)^3$
   (b) $\dfrac{(3y^3)^4}{6y^5}$

7. Classify each function given below, by circling the appropriate name or names.
   (a) $f(x) = 2^x$

   rational power exponential polynomial

   (b) $g(x) = x^2$

   exponential power root polynomial

   (c) $Q(t) = t^{1/2}$

   power root exponential rational

8. (a) Find a linear function passing through $(1, 6)$ and $(3, 24)$.

   (b) Find an exponential function passing through $(1, 6)$ and $(3, 24)$.

4. Trigonometric Functions

See Appendix D for a thorough review of trigonometry. It is especially important to be familiar with sine, cosine, and tangent. In particular, know the values of these three functions at the five standard angles in the first quadrant: $0, \pi/6, \pi/4, \pi/3, \pi/2$, and how to use these special values to find the values of the other trig functions in all four quadrants.

Exercises:

9. Fill in the following table, using the five standard angles in the first quadrant.

Angle, θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
deg	rad			

10. Evaluate the function at the given value.

    (a) $\cos(-120°) \;=$ ____________________

    (b) $\csc\left(\dfrac{3\pi}{4}\right) \;=$ ____________________

11. For each caption, fill in the blank with the trigonometric function.

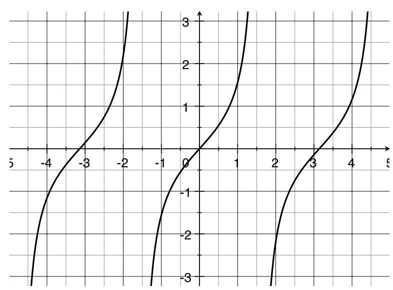

Figure 3: Graph of __________

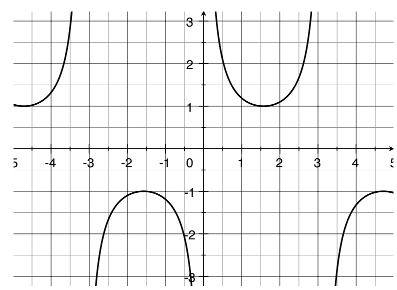

Figure 4: Graph of __________

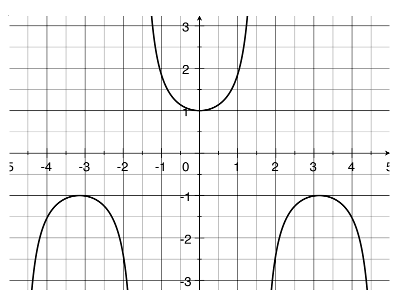

Figure 5: Graph of __________

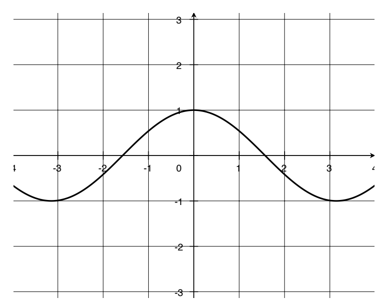

Figure 6: Graph of __________

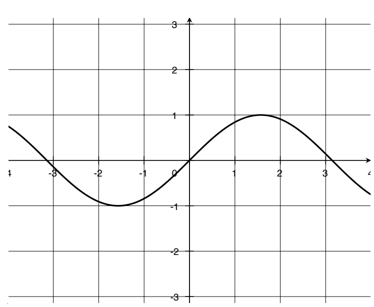

Figure 7: Graph of __________

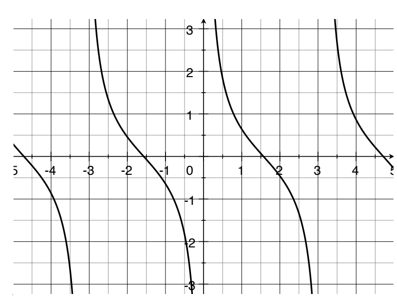

Figure 8: Graph of __________