**Name:** ______________________ **Section:** __________

**Names of collaborators:** ______________________

## Main Points:

1. Estimating limits from tables or graphs, right and left limits
2. Determining infinite limits, vertical asymptotes

## 1. The Limit of a Function

The formal mathematical notion of a *limit* is used to describe trends like the ones discussed in Section 2.1. Intuitively, if the values of $f(x)$ approach a specific number $L$, as $x$ approaches $a$ from either side, then that number $L$ is called the limit. (Read the first few paragraphs on page 87.)

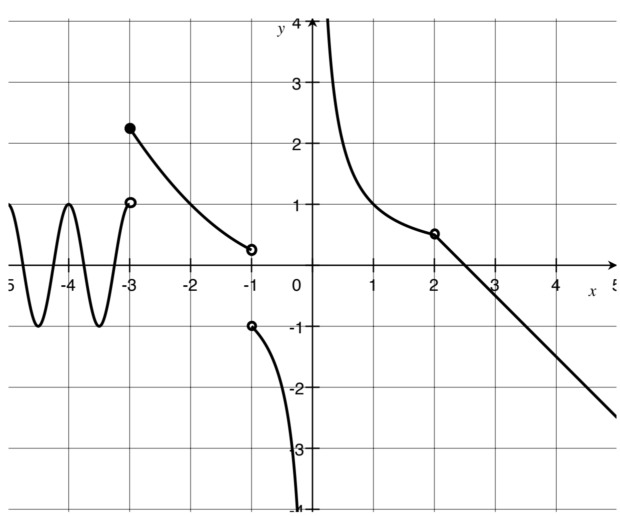
> **Definition** Suppose $f(x)$ is defined when $x$ is near the number $a$. (This means that $f$ is defined on some open interval that contains $a$, except possibly at $a$ itself.) Then we write $\lim_{x\to a} f(x) = L$ and say, "the limit of $f(x)$, as $x$ approaches $a$, equals $L$" if we can make the values of $f(x)$ arbitrarily close to $L$ (as close to $L$ as we like) by taking $x$ sufficiently close to $a$ (on either side of $a$) but not equal to $a$.

Similarly we can talk about the limit from the left and the limit from the right, if we only mean to discuss $a$ approaching $x$ from the left, or right, respectively. (See Example 7, page 92.)

When a function increases without bound (informally: the values "go to infinity") we use the infinity symbol ($\infty$) to denote the limit, even though the limit technically does not exist, because the values of the function to not approach a specific number. (See Example 8, page 93.)

**Exercises.**

1. Consider the function $f(x)$ in the graph below, and fill in the table to describe the behavior of the function at the specified points. Write "DNE" if the specified value does not exist.




| $a$ | $\lim_{x\to a^-} f(x)$ | $\lim_{x\to a^+} f(x)$ | $\lim_{x\to a} f(x)$ | $f(a)$ |
|---|---|---|---|---|
| $-3$ | | | | |
| $-1$ | | | | |
| $0$ | | | | |
| $2$ | | | | |
| $4$ | | | | |

2. Let $f(x) = \dfrac{\sin x}{x}$ and $a = 0$.

   (a) Use *Mathematica* to fill in the tables below, with numbers accurate to six digits.

| $x$ | $f(x)$ |
|---|---|
| 1 | |
| 0.1 | |
| 0.01 | |

| $x$ | $f(x)$ |
|---|---|
| $-1$ | |
| $-0.1$ | |
| $-0.01$ | |

   (b) Use your work in (a) to estimate the following limits, if they exist. If a limit does not exist, write "DNE".

$$\lim_{x\to 0^-} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 0^+} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 0} f(x) = \underline{\qquad\qquad}$$

   (c) What is $f(0)$? What does this tell you about what the graph looks like near $x = 0$?

3. Let $f(x) = \dfrac{x}{x-1}$ and $a = 1$.

   (a) Use *Mathematica* to fill in the tables below, with numbers accurate to six digits.

| $x$ | $f(x)$ |
|---|---|
| 2 | |
| 1.1 | |
| 1.01 | |
| 1.001 | |

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 0.9 | |
| 0.99 | |
| 0.999 | |

   (b) Use your work in (a) to estimate the following limits, if they exist. If a limit does not exist, write "DNE".

$$\lim_{x\to 1^-} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 1^+} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 1} f(x) = \underline{\qquad\qquad}$$

   (c) What is $f(1)$? What does this tell you about what the graph looks like near $x = 1$?

4. Let $f(x) = \dfrac{x^2-1}{x-1}$ and $a = 1$.

   (a) Use *Mathematica* to fill in the tables below, with numbers accurate to six digits.

| $x$ | $f(x)$ |
|---|---|
| 2 | |
| 1.1 | |
| 1.01 | |
| 1.001 | |

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 0.9 | |
| 0.99 | |
| 0.999 | |

   (b) Use your work in (a) to estimate the following limits, if they exist. If a limit does not exist, write "DNE".

$$\lim_{x\to 1^-} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 1^+} f(x) = \underline{\qquad\qquad} \qquad \lim_{x\to 1} f(x) = \underline{\qquad\qquad}$$

   (c) What is $f(1)$? What does this tell you about what the graph looks like near $x = 1$?

5. Look back at your work with the slope functions in the previous packet and write your conclusions using the limit notation:

   (a) For $f(x) = \sqrt{x}$, $a = 4$:

$$\lim_{h\to 0^-} D(h) = \underline{\qquad\qquad} \qquad \lim_{h\to 0^+} D(h) = \underline{\qquad\qquad} \qquad \lim_{h\to 0} D(h) = \underline{\qquad\qquad}$$

   (b) For $f(x) = |x^2-4|$, $a = 2$:

$$\lim_{h\to 0^-} D(h) = \underline{\qquad\qquad} \qquad \lim_{h\to 0^+} D(h) = \underline{\qquad\qquad} \qquad \lim_{h\to 0} D(h) = \underline{\qquad\qquad}$$

## 2. Determining Infinite Limits and Vertical Asymptotes from Formulas

We explored the concept of infinite limits above, using graphs and tables. We can also determine some infinite limits symbolically. (Read Examples 9 and 10 on page 95.) In stating vertical asymptotes, remember that the equation of a vertical line is of the form $x = a$, for some constant $a$.

**Exercises.**

6. Evaluate the following infinite limits symbolically:

   (a) $\displaystyle\lim_{x\to 5^-} \frac{6}{x-5}$

   (b) $\displaystyle\lim_{x\to 3} \frac{2-x^3}{(3-x)^2}$

   (c) $\displaystyle\lim_{x\to \pi^-} \csc x$

7. State the equations of the vertical asymptotes of the functions in the previous problem:

   (a) __________ (b) __________ (c) __________