

Name: _____

Section: _____

Names of collaborators: _____

Main Points:

1. Evaluating limits using limit laws
2. Evaluating limits using the Squeeze Theorem

1. Evaluating Limits Using Limit Laws and Other Theorems

So far, we have estimated limits by looking at graphs and tables, and we have discussed infinite limits, but we have not actually computed exact values for any finite limits. The good news is that the functions in our catalogue of functions are very well-behaved, and often we can evaluate limits simply by plugging in a number.

Read the eleven limit laws on pages 99-101 of the textbook. These limit laws tell us how to compute limits of combinations of functions, as long as we know the limits of the basic pieces making up the combination. In particular, a nice consequence is the **Direct Substitution Property** for polynomial and rational functions.

If f is a polynomial or rational function and a is in the domain of f , then

$$\lim_{x \rightarrow a} f(x) = f(a)$$

i.e. the limit can be evaluated by simply plugging in the number.

The catch is that the number a must be in the domain of f , and a rational function often has some numbers that are not in its domain. See Example 3 on page 102.

Exercises: Compute the following limits:

1. $\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$ (Hint: Start by factoring the numerator and the denominator.)

2. $\lim_{x \rightarrow -1} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$

3. $\lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

4. $\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$

Hint: Rationalize the numerator by multiplying numerator and denominator by the conjugate $\sqrt{1+h} + 1$. See Example 6, page 103.

For functions involving absolute value it is helpful to notice that taking the absolute value of a negative number is the same as multiplying by (-1) , e.g. $|-4| = (-1)(-4)$, so

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Sometimes it is necessary to compute left and right limits separately, for example for piecewise functions or functions involving an absolute value. If the left and right limits agree, say both are equal to a number L , then the limit itself is L . See Examples 7 and 8, page 104.

The **Squeeze Theorem** can also be useful for evaluating limits:

If $f(x) \leq g(x) \leq h(x)$ near a (except possibly at a), and

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$$

then,

$$\lim_{x \rightarrow a} g(x) = L$$

i.e. if the function $g(x)$ is sandwiched between two functions that have the same limit at a certain point, then the limit of $g(x)$ at that point must also be the same.

See Example 11, page 105.

Exercises:

5. Evaluate the limit: $\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x}$

6. Suppose that, for all x , $2x \leq g(x) \leq x^4 - x^2 + 2$. What can we say about $\lim_{x \rightarrow 1} g(x)$? (Hint: Use the Squeeze Theorem.)