Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Definition of limits at infinity and definition of horizontal asymptote
2. Theorem about rational powers of x
3. Infinite limits at infinity

1. The idea of limits at infinity

This section is about the "long term behavior" of functions, i.e. what happens as x gets really big (positive or negative). Sometimes the function will approach a specific number as x gets big. If a function f approaches a specific number L as x gets larger and larger (positive), we say that *the limit of $f(x)$ as x approaches infinity is L* and we write:

$$\lim_{x\to\infty} f(x) = L$$

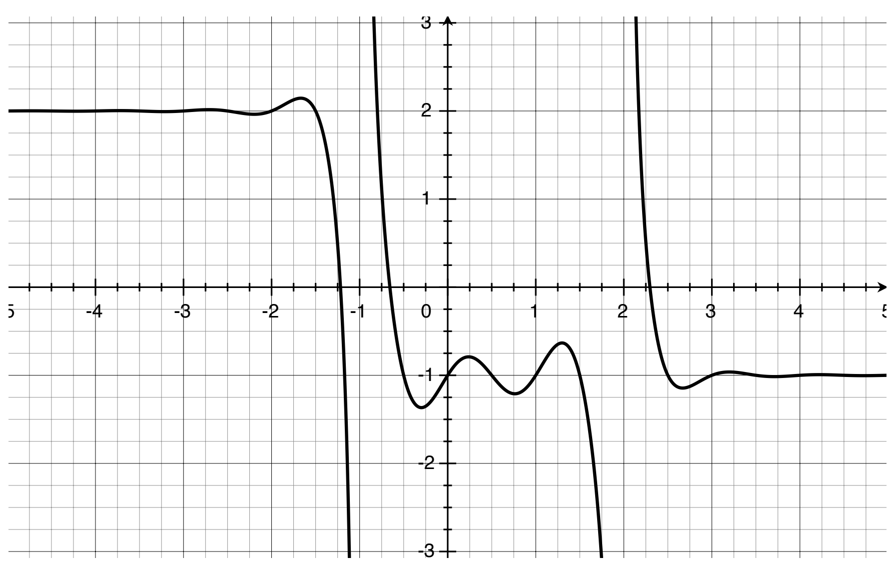
Similarly, if f approaches L as x becomes larger and larger negative, we write

$$\lim_{x\to-\infty} f(x) = L$$

In the first case, the graph of $f(x)$ flattens out on the right, and it approaches the line $y = L$. In the second case, the graph flattens out on the left, approaching the line $y = L$. In either case this line is called a *horizontal asymptote* of $f(x)$.

Exercises

1. Find the infinite limits, limits at infinity, and asymptotes for the function shown below. (See Example 1, page 132).



2. Consider the function $f(x) = \dfrac{x^2}{2^x}$.

   (a) Use *Mathematica* to fill in the table below, with numbers accurate to six digits.

x	$f(x)$	x	$f(x)$
0		7	
1		8	
2		9	
3		10	
4		20	
5		50	
6		100	

   (b) Based on your work in (a), estimate the value of the limit: $\lim_{x\to\infty} \dfrac{x^2}{2^x} \approx$ __________

2. Evaluating limits at infinity

The following theorem is helpful in evaluating infinite limits symbolically, particularly when dealing with rational functions.

Theorem: If r is a positive rational number, $\lim_{x\to\infty} \dfrac{1}{x^r} = 0$. Additionally, as long as x^r is *defined* for negative values of x, $\lim_{x\to-\infty} \dfrac{1}{x^r} = 0$.

The key to using this theorem with rational functions is to divide top and bottom by the highest power of x in the denominator. See Example 3, page 133.

Of course, it is also possible to have an infinite limit at infinity. (See page 136.) Many familiar functions have infinite limits at infinity, e.g. $\lim_{x\to\infty} x^2 = \infty$, and $\lim_{x\to-\infty} x^3 = -\infty$. See Example 9, page 136 and Example 11, page 137.

Exercises:

3. Evaluate the following limit: $\lim_{x\to\infty} \dfrac{3x+5}{x-4}$.

4. Evaluate the following limits:

   (a) $\displaystyle \lim_{t \to -\infty} \frac{t^2 + 2}{t^3 + t^2 - 1}$

   (b) $\displaystyle \lim_{t \to -\infty} \frac{t^3 + t^2 - 1}{t^2 + 2}$

5. Evaluate the following limit: $\displaystyle \lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}$.

   (Hint: when x is large, the bottom behaves like $3x$, since the $+1$ is small compared to the $9x^2$. So, instead of dividing top and bottom by x^2, divide top and bottom by x. See Example 4, page 134.)

6. Find the horizontal and vertical asymptotes of $f(x) = \dfrac{x^2 - 1}{x^2 - x - 2}$. Use limits to justify your work. Sketch a graph of the function that shows all the key features (asymptotes and other discontinuities.)