Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Definition of derivative as limit of difference quotients
2. Interpretation of derivative as slope of graph
3. Interpretation of derivative as instantaneous rate of change

1. Limit of Difference Quotients

Recall from Prep 2.1, that to estimate the slope of a tangent line, we use the slopes of secant lines. In particular, to estimate the slope of the tangent line to the graph of a function $f(x)$ at a point $x = a$ in the domain of f, the slope of the secant line through $P = (a, f(a))$ and $Q = (a + h, f(a + h))$ is:

$$D \;=\; \frac{\Delta y}{\Delta x} \;=\; \frac{f(a+h)\,-\,f(a)}{h}$$

This is called the *difference quotient*. The value of D depends on h.

The limit of $D(h)$ as h approaches zero (if the limit exists) is the slope of the tangent line. In the preparatory assignment on limits, you explored this idea numerically, by creating tables of D-values for smaller and smaller values of h.

Definition For a function $f(x)$ and a number a in its domain, the derivative of f at a, denoted $f'(a)$, is:

$$f'(a) \;=\; \lim_{h\to 0} D(h) \;=\; \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

provided that the limit exists.

Exercise

1. Look back at Prep 2.1 and Prep 2.2 to answer these questions.

   (a) If $f(x) = \sqrt{x}$ and $a = 4$, what is a good estimate for $f'(a)$?

   (b) If $f(x) = |x^2 - 4|$ and $a = 2$, what can you say about $f'(a)$?

2. Find an equation of the tangent line to the graph of $y = f(x)$ at $x = 5$ if $f(5) = -3$ and $f'(5) = 4$.

3. Suppose that $y = 4x - 5$ is an equation of the tangent line to the curve $y = f(x)$ at the point $x = 2$. Find $f(2)$ and $f'(2)$.

Having discussed how to evaluate limits symbolically (as in 2.3), we can determine the exact value for $f'(a)$, at least in some relatively simple examples. (See Example 4 on page 147.)

Exercise

4. Let $f(x) = \sqrt{x}$ and $a = 4$. Find the exact value of $f'(a)$ by evaluating the following limit symbolically as in Section 2.3:
$$\lim_{h \to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h}$$
(You will need to rationalize the numerator by multiplying numerator and denominator by the conjugate $\sqrt{4+h} + \sqrt{4}$. See Section 2.3, Example 6, page 103.)

2. Interpretation as Rates of Change

Recall that, in applications, we interpret slopes and difference quotients as rates of change. In particular, the difference quotient is an average rate of change. The limit of the difference quotient, the derivative, is the instantaneous rate of change.

In summary, we can interpret the difference quotient and the derivative in these ways:

difference quotient	derivative
slope of secant	slope of tangent
average velocity	instantaneous velocity
average rate of change	instantaneous rate of change

For examples of interpretating difference quotients and derivatives, see Examples 6 and 7, p 148-149.

Exercises

5. Look back at Exercise 3 from Prep 2.1. Estimate the instantaneous velocity of the buoy at $t = 0$. (Make sure to include units.)

6. Let $T(t)$ be the temperature (in °F) in Phoenix t hours after midnight on September 10, 2008. The table shows values of this function recorded every two hours.

t	0	2	4	6	8	10	12	14
T	82	75	74	75	84	90	93	94

   (a) Look back at your Exercise 4 from Prep 2.1, and give an estimate $T'(8)$.

   (b) What is the meaning of $T'(8)$? Use a complete sentence, and make sure to include units.

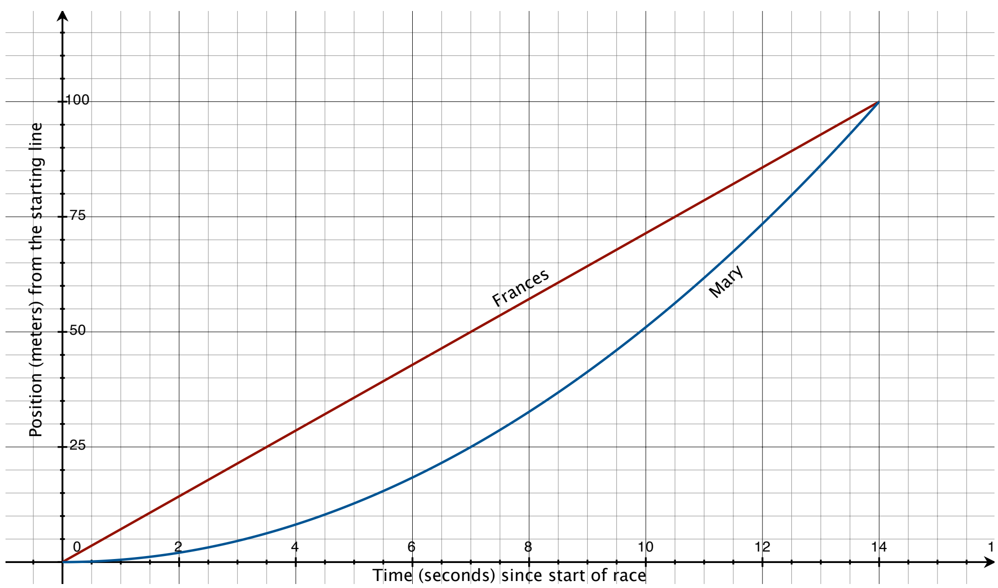
   (c) Use (b) to estimate the temperature at 9:00 am.

7. The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$.

   (a) What is the meaning of the statement $f'(5) = 100$? What are the units of $f'(5)$?

   (b) Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? Why?

8. Shown are graphs of the position functions of two runners, who run a 100 meter race.

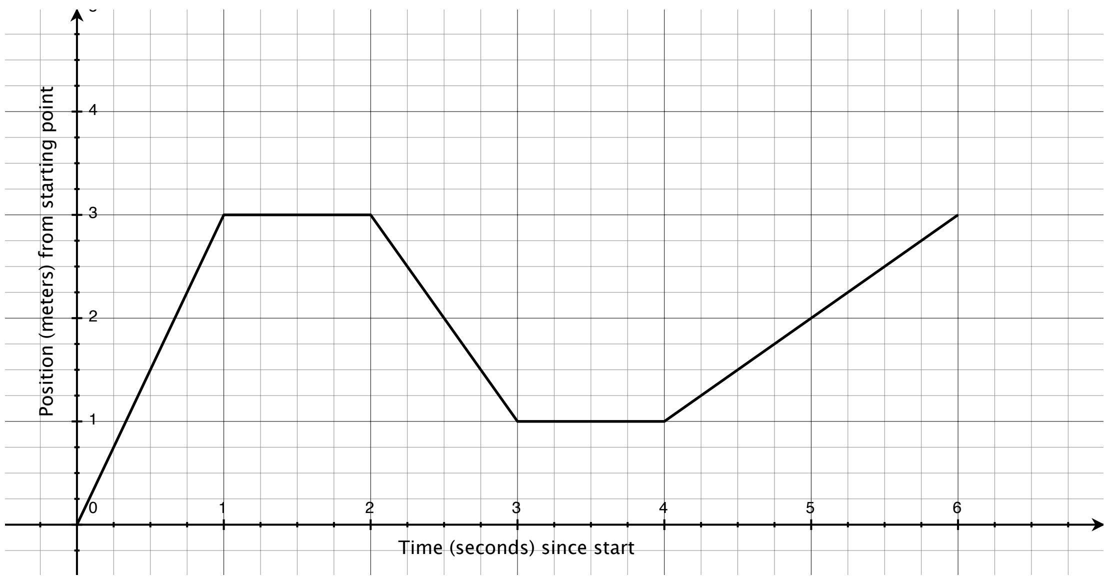


   (a) Describe and compare how the runners run the race. (In particular, who won the race?)

   (b) At what time is the distance between the runners the greatest? __________

   (c) At what time do they have the same velocity? __________

9. A duck starts by waddling north on 35E; the graph of its position function is shown.



(a) When is the duck waddling north? ____________________________

(b) Waddling south? ____________________________

(c) Standing still? ____________________________

(d) Draw a graph of the velocity function on the axes below.