

Name: _____

Section: _____

Names of collaborators: _____

Main Points:

1. Definition of derivative function as limit of difference quotients
2. Differentiability
3. Graph of derivative function
4. Higher order derivatives

1. Definition of Derivative Function

Definition: If we have a function $f(x)$ we can define a new function, the *derivative* of f to be:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. We call $f(x)$ an *antiderivative* for $f'(x)$.

Note: This is the same expression we had for $f'(a)$ in the previous section. The only difference is that in the previous section we thought about a and $f'(a)$ as fixed numbers, and now we're thinking of $f'(x)$ as a function depending on x .

For each x -value in the domain of $f(x)$, the slope of the tangent at that x -value is the value of $f'(x)$. Thus to obtain a graph of $f'(x)$ from $f(x)$ we plot the slopes of $f(x)$. In particular, if the graph of $f(x)$ has a horizontal tangent (slope of zero) at $x = a$, then $f'(a) = 0$, i.e. the graph of $f'(x)$ has an x -intercept at a . If $f(x)$ is increasing at $x = a$, the tangent line has a positive slope, so $f'(a)$ is positive. On the other hand, if $f(x)$ is decreasing at $x = a$, $f'(a)$ is negative. See Example 1, page 155.

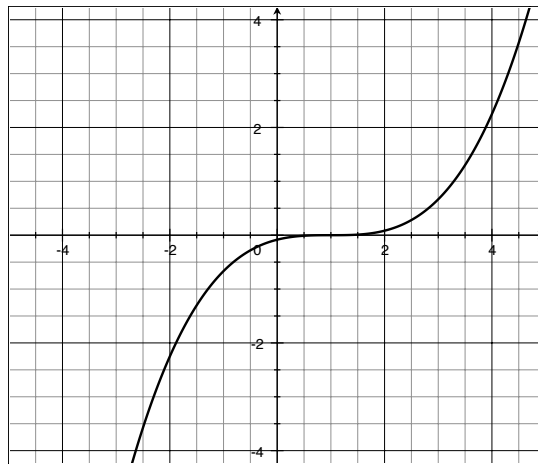
Exercises:

1. Sketch a graph of the function $f(x) = 5$. What is the derivative of this function? In general, what is the derivative of a constant function?

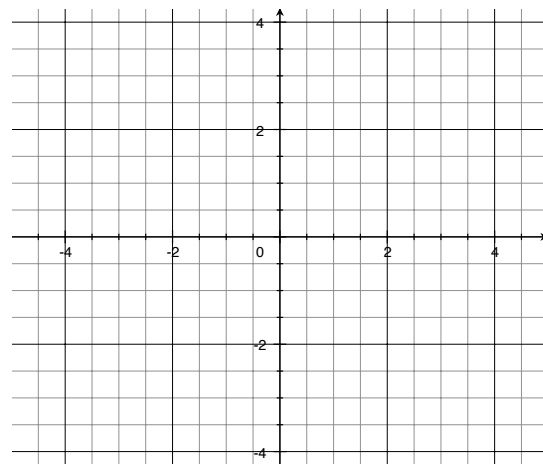
2. Sketch the graph of the function $f(x) = x$. What is the derivative of this function?

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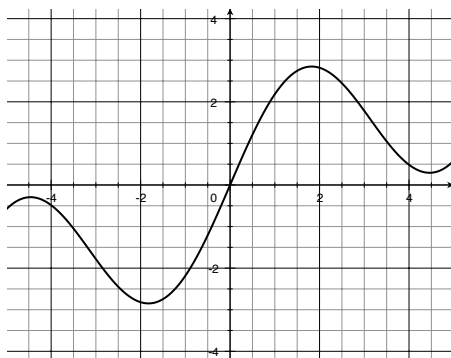
3. The graph of a function $f(x)$ is shown below. Estimate $f'(x)$ for each x -value in the table, by estimating the slope of the tangent line. Plot the points in your table to sketch a graph of the function $f'(x)$.



| x | $f'(x)$ |
|-----|---------|
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |



4. The graph of a function $f(x)$ is below. List the values in ascending order: $f'(-2)$, $f'(0)$, $f'(1)$, $f'(3)$. (Note: this means that the first number in your list should be the number that is less than all the other numbers. This will correspond to the steepest negative slope.)



2. Differentiability

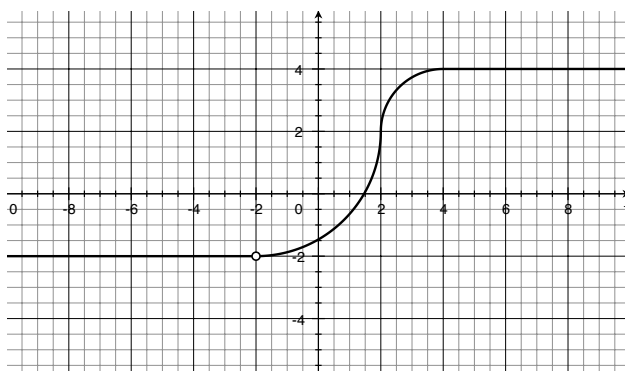
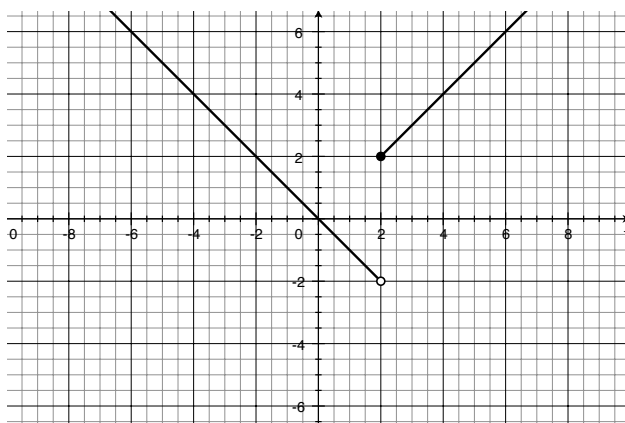
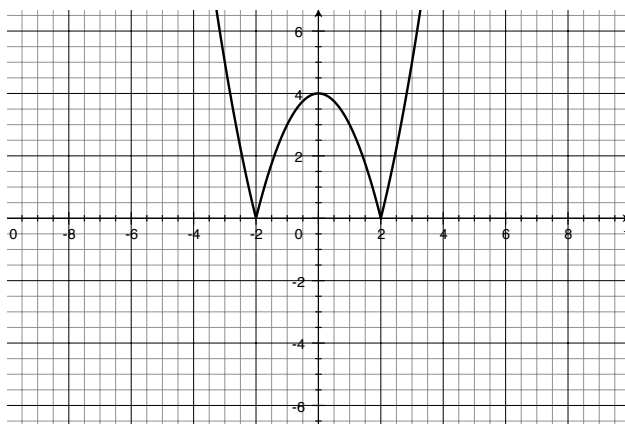
When a function $f(x)$ is not locally linear at a point $x = a$, i.e. when the graph of the function does not look linear even when you zoom in very close, the limit of difference quotients will not exist. (Recall the example of $f(x) = |x^2 - 4|$ and $a=2$.) In this case, we say that $f(x)$ is *not differentiable* at $x = a$. This means that the derivative function $f'(x)$ is *undefined* at $x = a$.

Here are some ways that $f(x)$ could fail to be differentiable (locally linear) at $x = a$: (1) f is discontinuous at a , (2) f has a corner at a , or (3) f has a vertical tangent at a . (See p 159-160.)

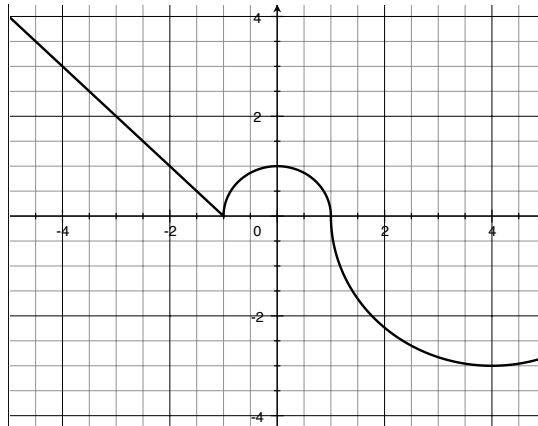
Note: If f is differentiable at a then it is necessarily continuous at a , but not vice versa.

Exercises:

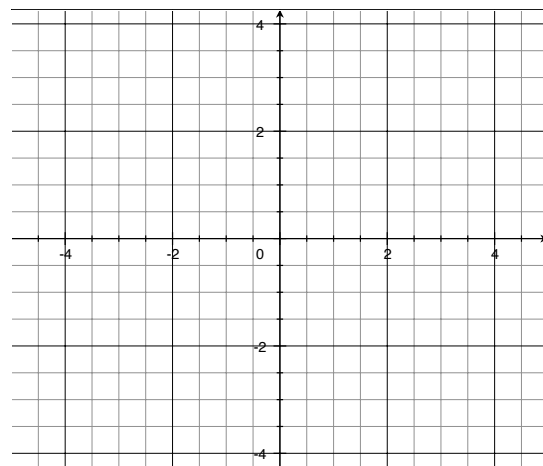
5. For each of the functions below, state, with reasons, the numbers at which the function is not differentiable.



6. The graph of a function $f(x)$ is shown below. Estimate $f'(x)$, if it exists, for each x -value in the table. If $f'(x)$ does not exist, write "DNE". Use your table to sketch a graph of the function $f'(x)$.



| x | $f'(x)$ |
|-----|---------|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |



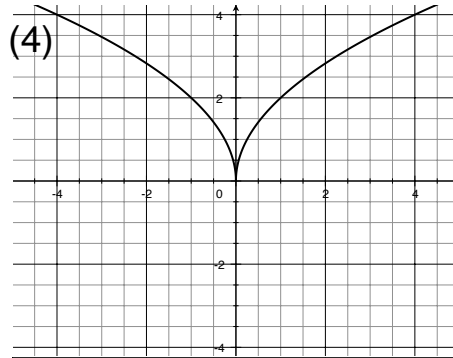
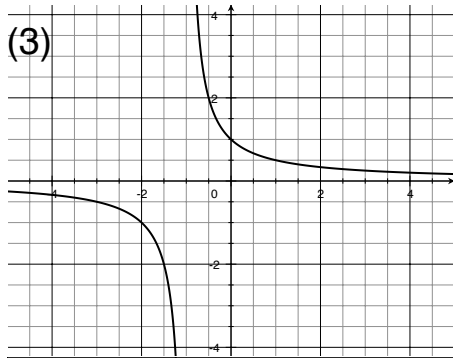
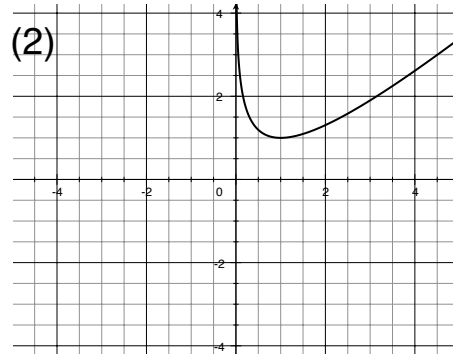
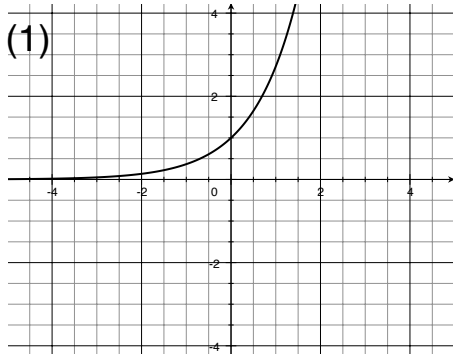
3. Graphs of Derivative Functions: Summary

| $f(x)$ | $f'(x)$ |
|-----------------------|--|
| horizontal tangent | x -intercept |
| discontinuity | undefined |
| corner | jump discontinuity or vertical asymptote |
| vertical tangent | vertical asymptote |
| increasing/decreasing | positive/negative |

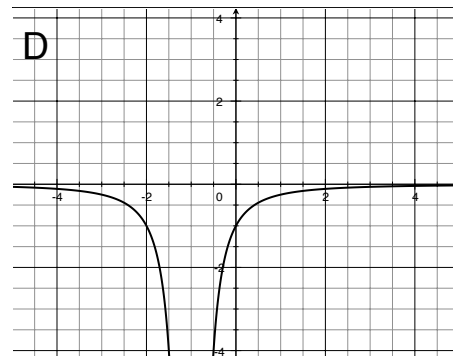
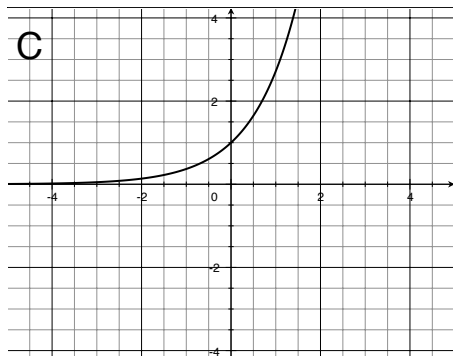
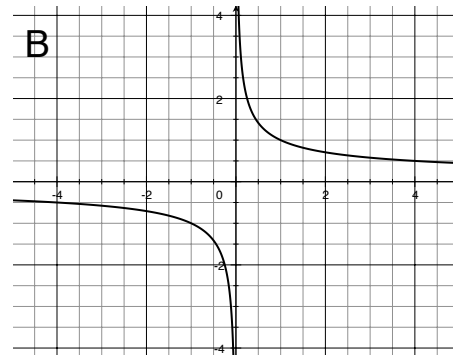
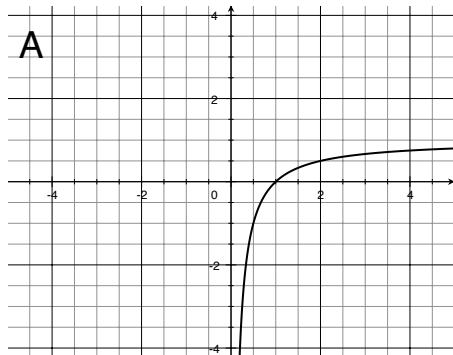
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7. Match the graph of the function with the graph of its derivative.

Graphs of four functions:



Graphs of the **derivatives** of the four functions:



4. Higher Order Derivatives

The derivative of $f'(x)$ is called the second derivative of $f(x)$ and is denoted $f''(x)$. Similarly, the derivative of $f''(x)$ is called the third derivative of $f(x)$ and is denoted $f'''(x)$. Because it is hard to see four or more primes, we do not denote the fourth or higher derivative with primes; instead the fourth derivative is denoted $f^{(4)}(x)$, the fifth derivative denote $f^{(5)}(x)$, etc.

Exercise:

8. The graphs of f , f' and f'' are depicted below. Identify and label each curve, and explain your choices.

