Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Finding a formula for the derivative function by evaluating the limit of difference quotients
2. Power Rule
3. Constant Multiple Rule, Sum Rule, Difference Rule
4. Derivative of e^x

1. Formulas for Derivative Functions

We can find a formula for $f'(x)$, given a formula for $f(x)$, using the definition:

$$f'(x) \quad = \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. (See 2.8, Example 2.)

Notation: If $y = f(x)$ the following expressions all mean the same thing:

$$f'(x) \qquad \frac{d}{dx} f(x) \qquad y' \qquad \frac{dy}{dx}$$

The different notations emphasize different things. For example, Leibnitz notaion $\frac{dy}{dx}$ emphasizes the relationship between the derivative and the difference quotient $D = \frac{\Delta y}{\Delta x}$, and the notation $\frac{d}{dx} f(x)$ emphasizes that we are doing something to (differentiating) the function $f(x)$.

Exercises:

1. Find a formula for the derivative of $f(x) = x^2$, using the definition of the derivative as the limit of difference quotients:

$$f'(x) \quad = \quad \lim_{h \to 0} \frac{(x+h)^2 \; - \; x^2}{h}$$

Hint: Start by expanding, i.e. "foiling," $(x + h)^2$.

2. Find a formula for the derivative of $f(x) = \sqrt{x} = x^{1/2}$, using the definition of the derivative as the limit of difference quotients:

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Hint: Rationalize the numerator by multiplying top and bottom by the conjugate: $\sqrt{x+h} + \sqrt{x}$.

2. Power Rule

Read pages 174-176, for the derivation of the power rule.

Exercises:

3. State the power rule (general version) as stated in the middle of page 176. Make sure to include the words, not just the formula.

4. Differentiate the power functions below using the power rule. Remember that $\frac{1}{x^a} = x^{-a}$ and $\sqrt[n]{x} = x^{1/n}$. (See Examples 1 and 2, pages 175 and 176.)

   (a) $f(x) = x^5$

   (b) $g(x) = \frac{1}{x}$

(c) $h(x) = \sqrt{x}$

(d) $F(x) = \sqrt[4]{x^3}$

3. Constant Multiples, Sums, and Differences of Functions

The constant multiple rule, sum rule, and difference rule tell us that differentiating functions that are constant multiples, sums, or differences of other functions works as simply as one could hope. (See pages 177-178.) For example,

$$\frac{d}{dx}\big(x^4 + 2x^3 - x\big) = \frac{d}{dx}\big(x^4\big) + 2\,\frac{d}{dx}\big(x^3\big) - \frac{d}{dx}\big(x\big) = 4x^3 + 2\big(3x^2\big) - 1 = 4x^3 + 6x^2 - 1$$

Exercises:

5. Differentiate the functions below. Keep in mind that sometimes you may need to do some algebraic simplification before using the differentiation rules.

   (a) $y = 3x^5 - \sqrt{x}$

   (b) $y = \dfrac{3}{x^2} - \pi$

   (c) $y = \dfrac{x^2 - 2\sqrt{x}}{x}$

4. Differentiating Exponential Functions

Read pages 179-180.

Exercises:

6. State the derivative of the natural exponential function, as given at the bottom of page 180.

7. Differentiate the functions below. Recall that $e^{x+y} = (e^x)(e^y)$.

   (a) $y = e^x + x^2$

   (b) $y = e^{x+1} + 1$

   (c) $y = e^x + x^e$

   (d) $y = e^{23} + e^{\pi}$