Name: _____   **Section:** _____

Names of collaborators: _____

1. The Product Rule and the Quotient Rule

Differentiating products and quotients of functions is not as straightforward as differentiating sums and differences of functions. Assuming that f and g are both differentiable functions, the rules below describe how to differentiate the product and quotient of f and g.

Product Rule: The derivative of a product of two functions is: the derivative of the first, times the second, plus the first times the derivative of the second:

$$(fg)' = f'g + fg'$$

Quotient Rule: The derivative of a quotient of two functions is: the derivative of the top, times the bottom, minus the top times the derivative of the bottom, all over the bottom squared:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Exercises:

1. Suppose $f(x) = x + 1$ and $g(x) = x^2 - 3$. Find the derivative of the product function fg in two ways:

   (a) using the Product Rule,

   (b) rewriting the product as a sum of power functions and then using the Power Rule.

   (c) Compare your two answers to make sure they are the same.

2. Find the derivative of $f(x) = \dfrac{x^4 - 5x^3 + \sqrt{x}}{x^2}$ in two ways:

   (a) using the Quotient Rule,

   (b) rewriting as a sum of power functions and then using the Power Rule.

   (c) Compare your answers to make sure they are the same.

3. Find the derivative of $g(x) = \sqrt[3]{x}\, e^x$.

4. Find the derivative of $y = 3x^7 e^{-x}$.

5. Find the derivative of $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$.

6. For $f(x) = \dfrac{x^2}{1 + 2x}$, find $f'(x)$ and $f''(x)$.

7. Suppose $f(x)$ is a differentiable function. Find the derivatives of the following functions:

(a) $y = x^2 f(x)$

(b) $y = \dfrac{f(x)}{x^2}$

(c) $y = \dfrac{x^2}{f(x)}$

(d) $y = \dfrac{x^2 f(x)}{1 + 5x}$

8. Find the derivatives, assuming that y is a function of x.

(a) $\dfrac{d}{dx} \sqrt{x}\, y$

(b) $\dfrac{d}{dx} x\,(x + y)$