Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. The derivative of sine and cosine

1. The Derivatives of Sine and Cosine

Skim through pages 191-194 to find the derivatives of sine and cosine and to find the two helpful limits used in the derivations.

Exercises:

2. State the derivatives and limits:

   (a) $\dfrac{d}{dx}\ \sin(x) \ =$

   (b) $\dfrac{d}{dx}\ \cos(x) \ =$

   (c) $\lim\limits_{x\to 0}\ \dfrac{\sin(x)}{x} \ =$

   (d) $\lim\limits_{x\to 0}\ \dfrac{\cos(x)-1}{x} \ =$

3. Differentiate. Simplify your answers using trigonometric identities, if possible.

   (a) $\dfrac{d}{dx}\ \sin(x)\cos(x) \ =$

   (b) $\dfrac{d}{dx}\ \dfrac{\sin(x)}{\cos(x)} \ =$

(c) $\dfrac{d}{dx}\ \dfrac{\cos(x)}{\sin(x)}\ =$

(d) $\dfrac{d}{dx}\ \dfrac{1}{\cos(x)}\ =$

(e) $\dfrac{d}{dx}\ \dfrac{1}{\sin(x)}\ =$

(f) $\dfrac{d}{dx}\ \dfrac{1+\sin x}{x+\cos x}\ =$

(g) $\dfrac{d}{dx}\ x\sin x\cos x\ =$

2. Derivatives of More Trig Functions

Formulas for the derivatives of tangent, cotangent, secant, and cosecant can be found using the derivatives of sine and cosine. The formulas can also be found on page 194.

Exercises:

4. State the derivatives:

   (a) $\dfrac{d}{dx}\ \tan(x)\ =$

   (b) $\dfrac{d}{dx}\ \cot(x)\ =$

   (c) $\dfrac{d}{dx}\ \sec(x)\ =$

   (d) $\dfrac{d}{dx}\ \csc(x)\ =$

5. Find the derivatives of the following functions:

   (a) $g(\theta) = 4\sec\theta + \tan\theta.$

   (b) $h(x) = x^{2/3}\csc x.$

   (c) $P(y) = \cot y + \cos(\pi)$

3. Limits involving sine and cosine

Recall the two helpful limits used to find the derivatives of sine and cosine:

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$$

Read Examples 5 and 6, page 196, to see how to use these two limits to evaluate similar limits.

Exercises

6. Evaluate the following limits:

   (a) $\lim_{x\to 0} \dfrac{\cos x - 1}{\sin x}$

   Hint: Rewrite $\dfrac{\cos x - 1}{\sin x} = \dfrac{\cos x - 1}{x} \cdot \dfrac{x}{\sin x}$.

   (b) $\lim_{x\to 0} \dfrac{\sin 4x}{\sin 6x}$

   Hint: Rewrite $\dfrac{\sin 4x}{\sin 6x} = \dfrac{4\sin 4x}{4} \cdot \dfrac{x}{x} \cdot \dfrac{6}{6\sin 6x} = \dfrac{4\sin 4x}{4x} \cdot \dfrac{6x}{6\sin 6x}$.

   (c) $\lim_{t\to 0} \dfrac{\sin^2 3t}{t^2}$