Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. The chain rule for derivatives of composite functions
2. Derivatives of exponentials

1. The Chain Rule

To differentiate a composite function like $F(x) = \cos^2(x)$ or $G(x) = \cos(x^2)$, use the chain rule.

Chain Rule: Take the derivative of the outer function, plug in the inner function, and multiply by the derivative of the inner function.

$$(f(g(x))' = f'(g(x)) \cdot g'(x)$$

Another way to write the chain rule is:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The derivatives of the example functions mentioned above are $F'(x) = (2\cos x) \cdot (\sin x) = 2\cos x \sin x$ and $G'(x) = (\sin(x^2)) \cdot (2x) = 2x \sin(x^2)$.

Exercises

1. Find the derivatives of the following functions:

   (a) $f(x) = (1 + x^4)^{2/3}$

   (b) $G(x) = \sqrt[3]{1 + \tan x}$

   (c) $y = 5 + \cos^3 x$

(d) $L(x) = 3x\ \sin(x^4)$

(e) $P(t) = e^{-t^2} + t + e^4$

(f) $y = \sin^2 5x$

(g) $y = e^{-5x} \cos 3x$

(h) $y = \sin 3x\ \cos 4x$

2. Suppose $f(x)$ is a differentiable function. Find the derivatives of the following functions:

   (a) $y = f(x^2)$

   (b) $y = \sqrt{f(x)}$

   (c) $y = \tan(f(x))$

3. Find the derivatives, assuming that y is a function of x.

   (a) $\frac{d}{dx}\ \sec(y)$

   (b) $\frac{d}{dx}\ y\, e^{x^2}$

2. Derivatives of Exponential Functions

We know that the derivative of the natural exponential $y = e^x$ is $\frac{dy}{dx} = e^x$. An exponential with any other base will have a slightly different derivative. See page 203 for an explanation of how we can use the chain rule to differentiate $y = a^x$ for an exponential function with any base $a > 0$.

Exercises:

4. State the derivative: $\dfrac{d}{dx}\, a^x \;=$

5. Differentiate the functions:

   (a) $f(x) = 2^x$

   (b) $g(x) = 100 \cdot (1/3)^x$

   (c) $h(x) = (\sqrt{2})^x + \sqrt{x} + \sqrt{2}$

   (d) $r(t) = t \cdot 4^t$

6. Find the slope of the tangent line to the curve at $(0, 1)$. Round your answers to 3 decimal places.

   (a) $y = 2^x$

   (b) $y = e^x$

   (c) $y = 3^x$