

Name: _____ Section: _____

Names of collaborators: _____

Main Points:

1. Derivatives of logarithmic functions
2. Derivatives of inverse trig functions

1. Derivatives of Logarithms

The procedure for finding derivatives of inverse functions will be outlined in the first exercise below.

Exercises

1. In this exercise we will find a formula for the derivative of $f(x) = \ln(x)$. Recall that, for $x > 0$,

$$f(x) = \ln(x) \quad \Longleftrightarrow \quad e^{f(x)} = x$$

We will differentiate both sides of the second equation: $\frac{d}{dx} e^{f(x)} = \frac{d}{dx} x$ to get a new equation.

(a) Use the chain rule to differentiate the left hand side: $\frac{d}{dx} e^{f(x)} =$

(b) On the other hand, the derivative of the right hand side is: $\frac{d}{dx} x =$

(c) So, if we differentiate both sides of the equation $e^{f(x)} = x$, what new equation do we get?

(d) Now solve for $f'(x)$, to get it on a side by itself.

(e) Now use the fact that $e^{f(x)} = x$ to get a formula for $f'(x)$ in terms of x .

(f) Thus we can conclude: For $x > 0$, $\frac{d}{dx} \ln(x) =$

2. Read page 218, and state the derivative of the logarithm base b :

$$\frac{d}{dx} \log_b(x) =$$

3. Find the derivatives:

(a) $\frac{d}{dx} \ln(x^2 + 1) =$

(b) $\frac{d}{dx} (\log_2(x) - \log_3(x)) =$

(c) $\frac{d}{dx} x^2 \ln(x) =$

Recall that it sometimes helps to rewrite a function before differentiating it. In particular, for functions involving logarithms, the laws of logarithms can be used to rewrite the function before differentiating.

Recall the laws of logarithms:

- $\log(ab) = \log a + \log b$
- $\log(a/b) = \log a - \log b$
- $\log(a^r) = r \log a$

Exercises

4. Use the laws of logarithms to rewrite the function; then differentiate.

(a) $f(x) = \ln(x^2 e^x)$

(b) $g(x) = \ln\left(\frac{\sqrt{x}}{x^7 \cos x}\right)$

2. Derivatives of Inverse Trigonometric Functions

We obtained a formula for the derivative of $f(x) = \ln x$ by writing $e^{f(x)} = x$ and differentiating both sides with respect to x . The same method works for obtaining formulas for the derivatives of the inverse trig functions.

Exercises

5. Read pages 213-214, and state the derivatives below:

(a) $\frac{d}{dx} \arcsin(x) =$

(b) $\frac{d}{dx} \arccos(x) =$

(c) $\frac{d}{dx} \arctan(x) =$

(d) $\frac{d}{dx} \operatorname{arccot}(x) =$

(e) $\frac{d}{dx} \operatorname{arcsec}(x) =$

(f) $\frac{d}{dx} \operatorname{arccsc}(x) =$

6. Find the derivatives:

(a) $\frac{d}{dx} \sqrt{\arctan x} =$

(b) $\frac{d}{dx} x \arcsin x =$

(c) $\frac{d}{dx} \arccos(x^2) =$