Name: ____________________ **Section:** ________

Names of collaborators: ____________________

Main Points:

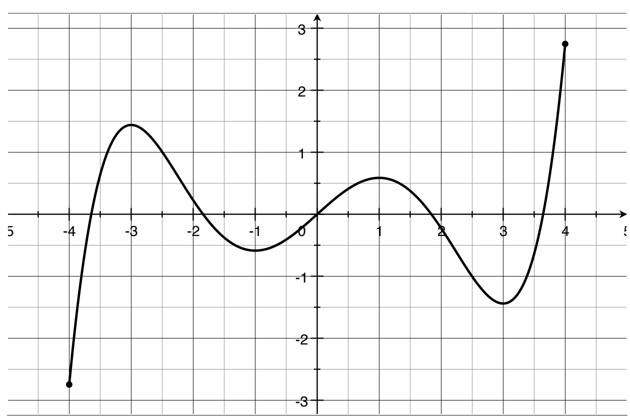
1. Notions of absolute and local max/min values
2. Definition of critical numbers and procedure for finding absolute max/min values on a closed interval

1. Absolute and local max/min values

Read page 274, about absolute/global and local maximum and minimum values. Note that max/min "value" refers to a y-value of a function. When we refer to the "location" of a max or min, this is the x-value at which the function has a max/min y-value.

Exercises:

1. Consider the function $f(x)$ whose domain is $D = [-4, 4]$ and whose graph is shown below.

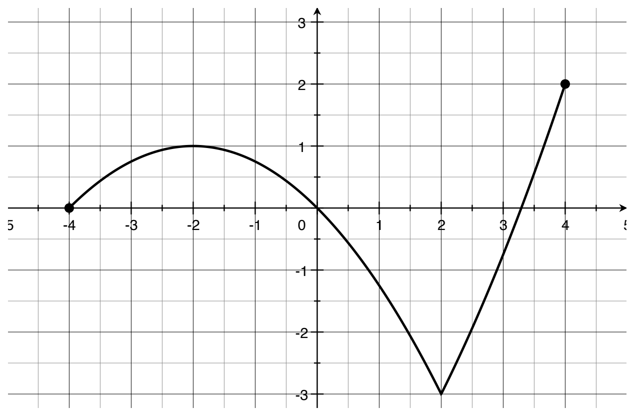


(a) Estimate the absolute maximum value of $f(x)$ and the absolute minimum value of $f(x)$. What are the locations of the absolute max and the absolute min?

(b) Estimate all the local maximum and minimum values of $f(x)$ and give their locations.

(c) For each point on the graph where a local maximum or minimum occurs, describe the derivative of $f(x)$ at that point.

2. Consider the function $g(x)$ whose domain is $D = [-4, 4]$ and whose graph is shown below.



   (a) Estimate the absolute maximum value of $g(x)$ and the absolute minimum value of $f(x)$. What are the locations of the absolute max and the absolute min?

   (b) Estimate all the local maximum and minimum values of $g(x)$ and give their locations.

   (c) For each point on the graph where a local maximum or minimum occurs, describe the derivative of $g(x)$ at that point.

3. Read Examples 1-4 on page 275. Then answer the True/False questions below. Use Examples 1-4 to back up your claims.

   (a) If a function has an absolute maximum value, there can be more than one point on the graph where the maximum value is achieved.

   (b) Every function must have an absolute maximum value and an absolute minimum value.

   (c) Every absolute maximum is also a local maximum.

4. Sketch a graph of the function by hand, and use your sketch to find the absolute and local max and min values of the function, if they exist.

   (a) $h(x) = 2^x$ on the domain $-1 \le x \le 1$

   (b) $\ell(x) = x^2 - 2$ on the domain $-1 < x < 1$

2. Finding absolute max/min values on a closed interval

The Extreme Value Theorem on page 275 says that every continuous function on a closed interval has an absolute max value and an absolute min value. Further, the absolute max/min values must either occur at a local max/min point or at an endpoint.

Exercise:

5. (a) Look back at the function $f(x)$ in Exercise 1 on this packet. Does the absolute maximum value occur at a local max/min point or at an endpoint? What about the absolute minimum?

   (b) Look back at the function $g(x)$ in Exercise 2 on this packet. Does the absolute maximum value occur at a local max/min point or at an endpoint? What about the absolute minimum?

If we want to find the absolute max/min values of a continuous function on a closed interval, we must look at endpoints and local max/min points. But how do we find local max/min points if we do not have a graph of the function? So-called "critical numbers" are important for finding the locations of these local max/min values.

Definition. A *critical number* for a function $f(x)$ is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.

Exercise:

6. (a) State the critical numbers for the function $f(x)$ in Exercise 1.

   (b) State the critical numbers for the function $g(x)$ in Exercise 2.

Theorem 7 (page 278) says that local max/min values occur only at critical numbers. This means that critical numbers are *candidates for locations* of local max/min values.

In particular, this means that the absolute max/min values of a continuous function on a closed interval must lie either at endpoints or critical numbers. This insight is the reason that the Closed Interval Method (page 278) is guaranteed to find the absolute max/min values of a continuous function on a closed interval.

Exercise:

7. Consider the function $f(x) = x^3 - 3x + 1$.

   (a) Find the critical numbers of $f(x)$.

   (b) Use the Closed Interval Method (page 278) to find the absolute max/min values of $f(x)$ on the interval $[0, 3]$.