Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Finding absolute max/min values on a closed interval
2. Rolle's Theorem and the Mean Value Theorem

1. Critical numbers and finding absolute max/min values

Recall that a critical number for a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist. By Theorem 7 in Section 4.1, every local max/min value occurs at a critical number, so critical numbers are candidates for local max/min values.

Exercises

1. Find the critical numbers of $g(x) = x\sqrt{1-x^2}$.

2. Find the critical numbers of $f(x) = |2x+1|$.

   Hint: Remember that $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$.

Recall that the Extreme Value Theorem on page 275 says that every continuous function on a closed interval has an absolute max value and an absolute min value. Further, the absolute max/min values must either occur at a critical number or at an endpoint. See page 278 for a description of The Closed Interval Method for finding absolute max/min values.

Exercises

3. Find the absolute maximum of the function on the given interval.

   (a) $f(x) = x + \frac{1}{x}$; $[0.2, 4]$

   (b) $g(x) = \sqrt{4 - x^2}$; $[-1, 1]$

3. Rolle's theorem and the MVT

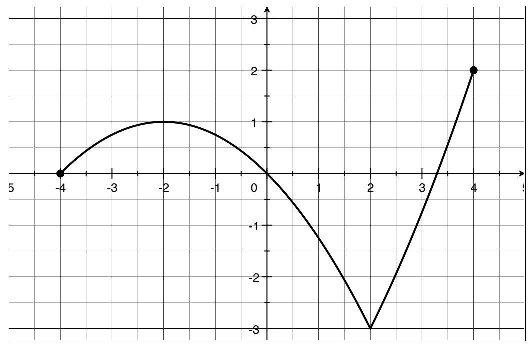
Rolle's Theorem: Suppose f is a function satisfying the following three conditions:

- f is continuous on a closed interval $[a, b]$
- f is differentiable on the open interval (a, b)
- $f(a) = f(b)$

Then we can conclude: There is a number c in (a, b) such that $f'(c) = 0$.

Exercise:

4. Consider the function $g(x)$ in depicted below.



(a) First we will check to see if $g(x)$ satisfies the hypotheses of Rolle's Theorem on $[-4, 0]$:

i. Is $g(x)$ continuous on $[-4, 0]$?

ii. Is $g(x)$ differentiable on $(-4, 0)$?

iii. Is $g(-4) = g(0)$?

(b) Now we will check to see if $g(x)$ satisfies the conclusion of Rolle's Theorem on $[-4, 0]$. Is there a number c between -4 and 0 such that $g'(c) = 0$? (If so, what is it?)

5. Verify that the function $f(x) = x^2 - 2x + 1$ satisfies the three hypotheses of Rolle's Theorem on the interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

Mean Value Theorem: Suppose f is a function satisfying the following two conditions:

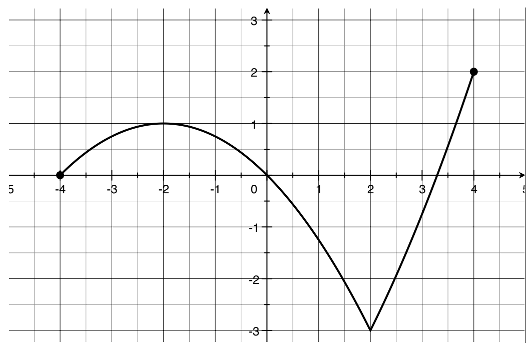
- f is continuous on a closed interval $[a, b]$
- f is differentiable on the open interval (a, b)

Then we can conclude: There is a number c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{or equivalently,} \qquad f(b) - f(a) = f'(c)(b - a)$$

To put a physical interpretation on this, think of driving a car. I could find my average speed by dividing the distance I traveled by the time it took me. The mean (meaning average!) value theorem says that there was (at least one) moment during my trip when my speed was exactly equal to my average speed.

6. Again, consider the function $g(x)$ whose graph is shown below.



   (a) First we will check to see if $g(x)$ satisfies the hypotheses of the Mean Value Theorem on $[-4, 2]$:
      i. Is $g(x)$ continuous on $[-4, 2]$?

      ii. Is $g(x)$ differentiable on $(-4, 2)$?

   (b) Now we will check to see if $g(x)$ satisfies the conclusion of the Mean Value Theorem on $[-4, 2]$.
      i. Sketch the secant line through the points $(-4, 0)$ and $(2, -3)$ on the graph above.
      ii. Is there a number c between -4 and 2 such that tangent line at $x = c$ is parallel to the secant line above? If so, estimate the value of c, and sketch the tangent on the graph above.

7. Verify that the function $f(x) = x^2 - 2x + 1$ satisfies the hypotheses of the MVT on the interval $[1, 3]$. Then find a number c that satisfies the conclusion of the MVT.