**Math 113, Prep 4.3: Derivatives and Shape of a Graph**

**Name:** ______________________________ **Section:** ________

**Names of collaborators:** ______________________________

## 1. Overview

**Increasing and Decreasing**: The *first* derivative gives increasing/decreasing information about the original function. (See p 290.)

$f'(x)$ *positive* $\longrightarrow$ slope of the tangent is positive $\longrightarrow$ $f(x)$ is ______________

$f'(x)$ *negative* $\longrightarrow$ slope of the tangent is negative $\longrightarrow$ $f(x)$ is ______________

The only places where $f$ can switch from increasing to decreasing are when $f'(x) = 0$ or $f'(x)$ DNE.

**Note**: Watch out, these numbers are not necessarily critical numbers, because critical numbers *have to be in the domain of* $f(x)$! For example $f(x) = \frac{1}{x^2}$ switches from increasing to decreasing at $x = 0$, but $x = 0$ is *not* a critical number because it is *not* in the domain of $f$.

**Local Maxima and Minima**: Remember from 4.1 that critical numbers are the only possibilities where local max/min may occur. (A local max/min surely must occur at a place in the domain where $f$ switches from increasing to decreasing or decreasing to increasing.) So we find the places where local max/min occur by checking each critical number $c$. (See p 291.)

$f'(x)$ negative to the left of $c$, $f'(x)$ positive to the right of $c$ $\longrightarrow$ local ________ at $c$

$f'(x)$ positive to the left of $c$, $f'(x)$ negative to the right of $c$ $\longrightarrow$ local ________ at $c$

If the sign of the derivative is the *same* on both sides of $c$, then there is neither a local min nor a local max at $c$. This way of checking the critical numbers is called the *first derivative test*.

**Concavity**: The *second* derivative gives concavity information about the original function. (See p 293.)

1. $f''(x)$ *positive* $\longrightarrow$ $f(x)$ is concave __________
2. $f''(x)$ *negative* $\longrightarrow$ $f(x)$ is concave __________

The only places where $f$ can switch concavity are when $f''(x) = 0$ or $f''(x)$ DNE.

**Inflection Points**: A point $(x, y)$ on the graph of $f(x)$ is called an *inflection point* if $f$ is continuous at $x$ and switches concavity at $x$. (Note that an inflection point is a *point* with an $x$-value and a $y$-value.)

**Note**: Just because a function switches concavity at $x$, that does not mean it will have an inflection point there. For example, $f(x) = \frac{1}{x}$, $f(x)$ switches concavity at $x = 0$, but $f(x)$ is undefined at $x = 0$, so there is no inflection point there.

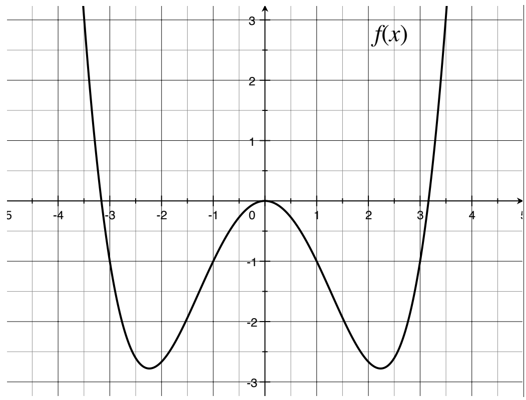
**Local Maxima and Minima Revisited**: Another way to check a critical number to see if a local max/min occurs there, is by checking concavity instead of increasing/decreasing. This is called the *second derivative test*. You can do this as long as $f''(c)$ exists. (See p 295.)

$f''(c)$ positive $\longrightarrow$ $f$ concave *up* at $c$ $\longrightarrow$ local ________ at $c$

$f''(c)$ negative $\longrightarrow$ $f$ concave *down* at $c$ $\longrightarrow$ local ________ at $c$

## 2. Exercises

1. Consider a function $f(x)$ whose graph is below.




   (a) On what intervals is $f(x)$ increasing?

   (b) State the local minimum and local maximum values of $f$, along with their locations.

   (c) On what intervals is $f(x)$ concave upward?

   (d) What are the of the inflection points of $f(x)$? (Make sure to include both $x$ and $y$-coordinates.)

2. Consider a function $F(x)$ whose *derivative* is graphed above, in Exercise 1, i.e. $F'(x) = f(x)$.

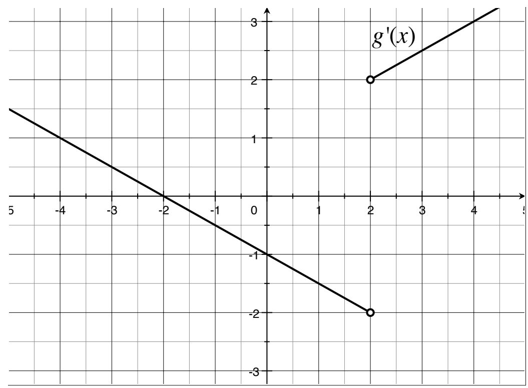
   (a) On what intervals is $F(x)$ increasing? Explain.

   (b) At what values of $x$ does $F(x)$ have a local maximum or minimum? Explain.

   (c) On what intervals is $F(x)$ concave upward? Explain.

   (d) What are the $x$-coordinates of the inflection points of $F(x)$? Why?

3. Suppose $g(x)$ is a continuous function whose derivative is shown below.



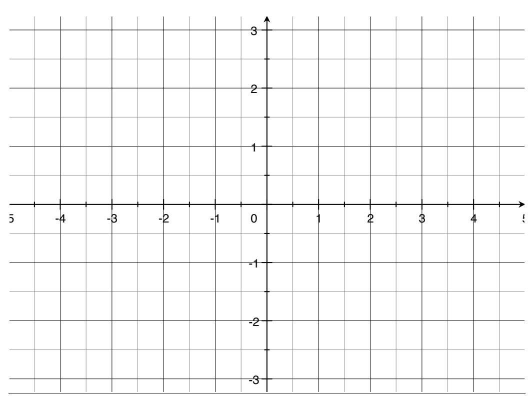

(a) On what intervals is $g(x)$ increasing? Decreasing?

(b) At what values of $x$ does $g(x)$ have a local maximum? Local minimum?

(c) On what intervals is $g(x)$ concave upward? Concave downward?

(d) What are the $x$-coordinates of the inflection points of $g(x)$?

(e) Assuming that $g(0) = 0$, sketch a graph of $g(x)$ on the axes provided.



4. Consider $f(x) = 5 - 3x^2 + x^3$. Find the intervals of increase and decrease, local maximum and minimum values, the intervals of concavity, and the inflection points. (See Example 6, p 295.)

5. Consider $f(x) = \sin x + \cos x$. Find the intervals of increase and decrease, local maximum and minimum values, the intervals of concavity, and the inflection points.