Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Antiderivatives by recognition
2. Initial value problems

1. Antiderivatives by recognition

If $g(x)$ is the derivative of $f(x)$, i.e. $f'(x) = g(x)$, then we say that $f(x)$ is *an antiderivative* for $g(x)$. For example, consider $f(x) = x^2 + 3$ and $g(x) = 2x$. In this case, clearly, $g(x)$ is the derivative of $f(x)$, so $f(x)$ is an antiderivative of $g(x)$.

Notice that $h(x) = x^2 + 7$ is also an antiderivative of $g(x)$, as is $p(x) = x^2 - 11$. In fact, any function of the form $x^2 + C$, where C is a fixed real number, is an antiderivative of $g(x)$. This is the most general form of the antiderivative, and it represents an entire family of functions.

Exercises.

1. Find an antiderivative for each of the given functions. (The key is to recognize the given function as the derivative of a familiar function.) Check your answer by differentiating.

   (a) $g(x) = 4x^3$

   (b) $j(\theta) = \cos\theta + 9$

   (c) $P(t) = \sec t \tan t$

   (d) $f(x) = e^x - \frac{1}{x}$

   (e) $G(x) = \ln(3) \cdot 3^x$

2. Write the most general form of the antiderivative for each of the given functions.

   (a) $T(u) = u^2 + u + 1$

   (b) $B(t) = \sec^2 t$

   (c) $f(x) = 2e^{2x}$

   (d) $\ell(x) = \dfrac{1}{1+x^2}$

One pattern worth noticing is that the power rule for differentiation can be reversed: since $\frac{d}{dx}\, x^a = a\, x^{a-1}$ as long as $a \neq 0$, an antiderivative for $a\, x^{a-1}$ is x^a, as long as $a \neq 0$. We can rewrite this from the perspective of antidifferentiating:

> **Reverse Power Rule**: For the power function x^p, with $p \neq -1$, the most general form of the antiderivative is $\frac{1}{p+1}\, x^{p+1} + C$.

The antiderivative of x^{-1} is $\ln|x|$, since $\frac{d}{dx} \ln|x| = \frac{1}{x}$. (See Example 6, page 220, in Section 3.6.)

Exercises

3. Find an antiderivative for each of the given functions.

   (a) $Q(t) = t^6 - t^{-6}$

   (b) $g(x) = \sqrt{x} + 1$

(c) $v(t) \;=\; t(t - \frac{1}{t})$ Hint: Distribute.

(d) $r(x) \;=\; \sec x(\tan x + \sec x)$

(e) $H(x) \;=\; \dfrac{e^x + e^{2x}}{e^x}$ Hint: Rewrite as the sum of two fractions and simplify.

2. Initial value problems

Given a velocity function for a particle, we may wonder if we can find the position function. We know that the velocity is the derivative of the position, so, therefore, the position should be an antiderivative of the velocity. However, unless we have some additional information, it is impossible to pick out which antiderivative in the family is the correct position function. Knowing the initial position of the particle is sufficient information to determine which antiderivative is the position function.

See Examples 3, 4, 6, and 7.

Exercises

4. Suppose that the velocity of a particle, in meters per second, is given by the function $v(t) = 3\cos t$ and that the initial position of the particle is $s_0 = 1$ meters. Find the position function $s(t)$.

   Hint: First find the most general form of the antiderivative for $v(t)$; then use the fact that $s(0) = 1$ to solve for the constant.

5. Suppose that the acceleration of a particle, in feet per seconds squared, is given by the function $a(t) = -16$.

   (a) Given that the initial velocity of the particle is $v_0 = 5$ feet per second, find the velocity function.

   (b) Given that the initial position of the particle is $s_0 = 2$ feet, find the position function.

6. **Challenge:** Suppose the velocity of a particle is shown in the graph below and that the initial position of the particle is $s_0 = 0$. Sketch a rough graph of the position of the particle.