Name: ______________________________ **Section:** ________

Names of collaborators: ____________________________________

Main Points:

1. Estimating accumulated (net) change over an interval
2. Area under a curve

1. Estimating accumulated change

If a quantity Q changes at a constant rate r, with respect to time, then the net change ΔQ in Q over the time interval Δt is exactly: $\Delta Q = r \cdot \Delta t$.

Of course, if Q does not change at a constant rate, this is will not work. Suppose that the instantaneous rate of change of Q, with respect to time, is given by $r(t)$. Then we can *estimate* the change in Q over a short time interval $\Delta t = t_2 - t_1$ by: $\Delta Q \approx r'(t_1) \cdot \Delta t$ or $\Delta Q \approx r(t_2) \cdot \Delta t$. We have done this in the section on linear approximation.

If we wish to estimate the change in Q over a longer time interval, we could break the time interval into short pieces, estimate the change in Q over each short time interval, and then add up these small changes. For an example with distance and velocity, see 5.1, Example 4.

Exercises

1. The rate of change of the world's population, in millions of people per year, is given below.

Year	1950	1960	1970	1980	1990	2000
Rate of change	37	41	78	77	86	79

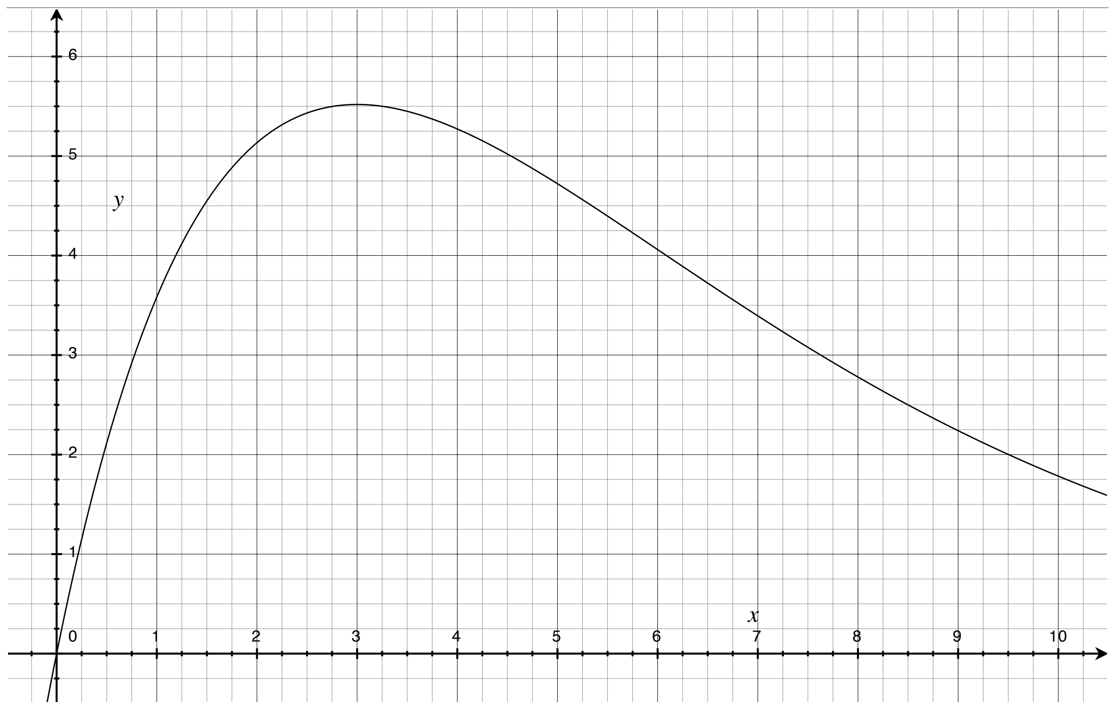
(a) Use this data to estimate the total change in the world's population between 1950 and 2000.

(b) The world population was 2555 million people in 1950 and 6085 million people in 2000. Calculate the true value of the total change in the population. How does this compare with your estimate above?

If we have a graph of the rate $r(t)$, then we can see that the multiplication of $r(t_1) \cdot \Delta t$ represents the area of a rectangle of height $r(t_1)$ and width Δt. Thus estimating net change can be understood as estimating the area under a curve using rectangles, whose height is determined by the graph of the function.

Exercise:

2. The velocity (ft/sec) of an object over time (sec) is shown in the graph below.



(a) Estimate the total distance the object traveled between $t = 0$ and $t = 10$, using velocity data every five seconds.

(b) Estimate the total distance the object traveled between $t = 0$ and $t = 10$, using velocity data every two seconds.

3. (See 5.1, Example 1.) Consider the region under the curve $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$.

   (a) Estimate the area using 5 approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is this an over-estimate or under-estimate?

   (b) Estimate the area using 5 approximating rectangles and *left* endpoints. Sketch the graph and the rectangles. Is this an over-estimate or under-estimate?

   (c) Estimate the area using 5 approximating rectangles and *midpoints*. (See Example 3b in the text.) You may use a calculator for this computation. Sketch the graph and the rectangles. Can you tell if this an over-estimate or an under-estimate?