

Name: _____

Section: _____

Names of collaborators: _____

Main Points:

1. Intervals of increase/decrease, local max/min points, intervals of concavity, inflection points
2. Absolute max/min points, optimization
3. Rolle's theorem, MVT
4. L'Hospital's rule for evaluating limits
5. Estimating accumulated net change over an interval; approximating area under curve with rectangles
6. Exact accumulated net change over an interval; definite integral
7. Area-so-far functions, antiderivatives, FTC, evaluating definite integrals using antiderivatives

Practice Problems:

1. Fill in the blank.

The absolute maximum value of a continuous function on a closed interval will always be obtained either at _____ or at _____. Local maximum values are always attained at _____.

If f is a differentiable function such that $f(2) = f(5)$, then Rolle's Theorem implies that, for some number c between 2 and 5, _____.

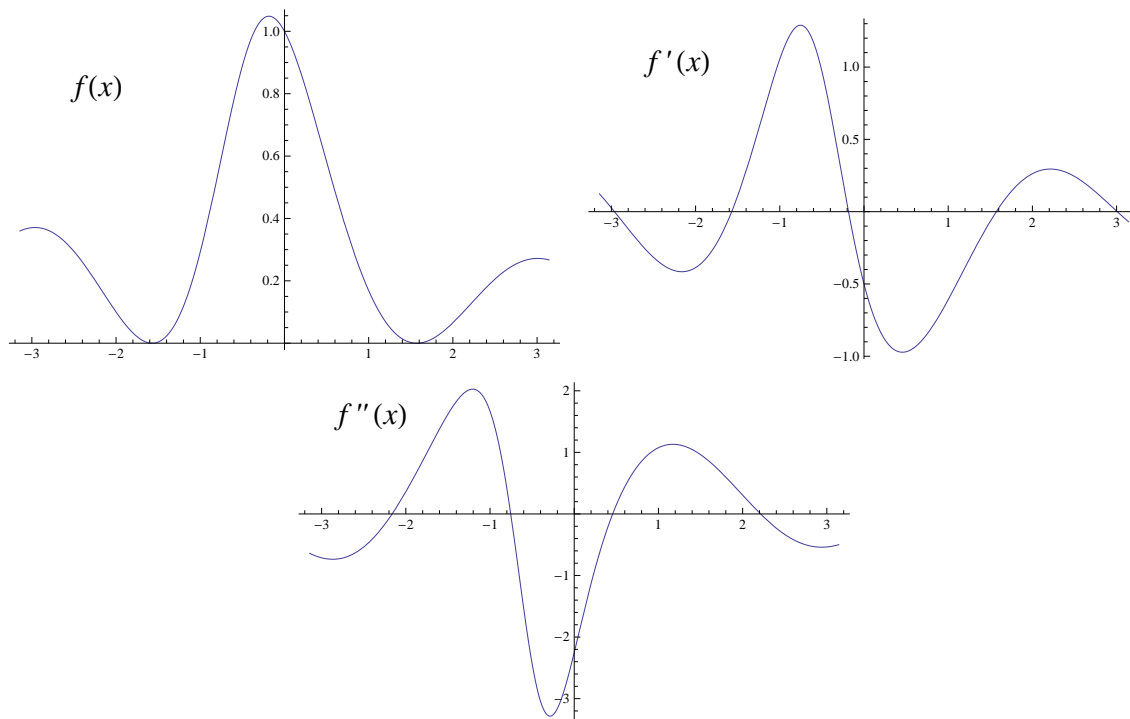
When $C''(s)$ is _____, $C'(s)$ is decreasing, and $C(s)$ is _____.

True or False: If $\lim_{x \rightarrow -1} f(x) = \infty$ and $\lim_{x \rightarrow -1} g(x) = \infty$, then $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)} = 1$. _____

If $\int_5^{10} f(x) dx = 7$ and $\int_0^{10} f(x) dx = 3$, then $\int_0^5 f(x) dx =$ _____.

If $g(x) = \int_1^x \arctan(t) dt$, then $A'(x) =$ _____.

True or False: If $F'(x) = G'(x)$, then $F(x) = G(x)$. _____

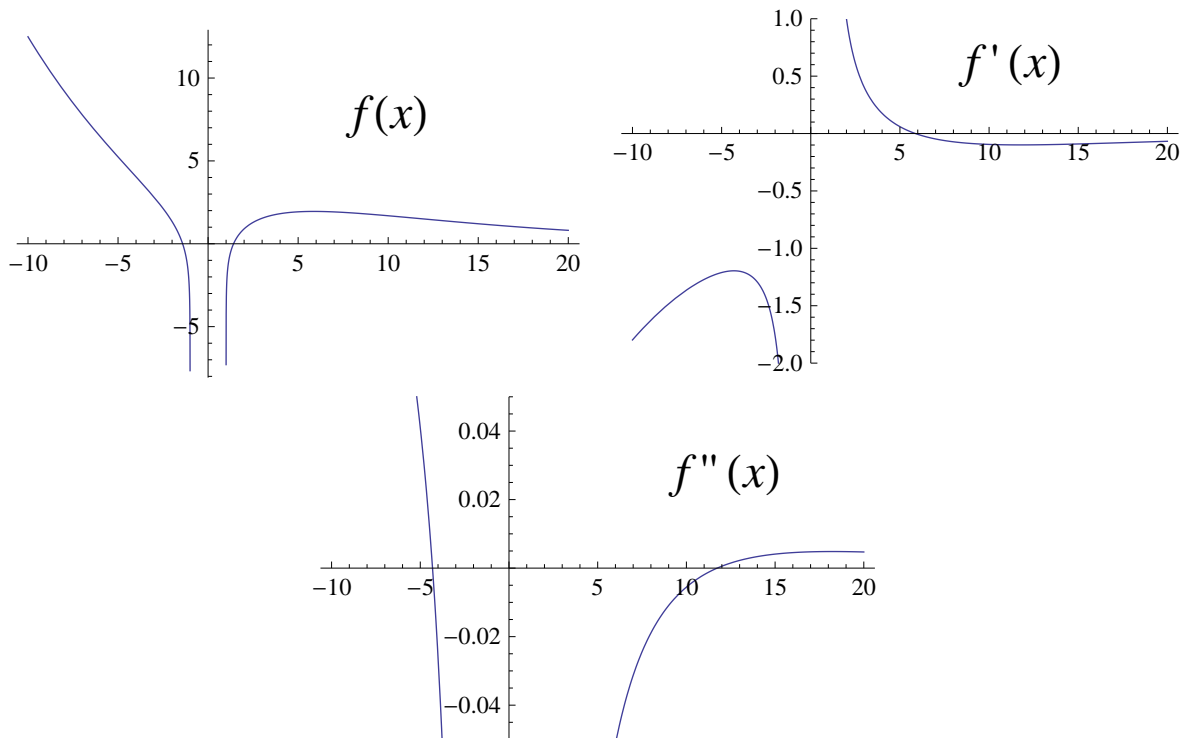


2. Graphs of f , f' , and f'' are shown above.

- (a) Estimate the x -coordinates of the local maximum points of f . Your answer should be within 0.2 units of the actual value.

- (b) Estimate the x -coordinates of the local minimum points of f . Your answer should be within 0.2 units of the actual value.

- (c) Estimate the x -coordinates of the inflection points of f . Your answer should be within 0.2 units of the actual value.



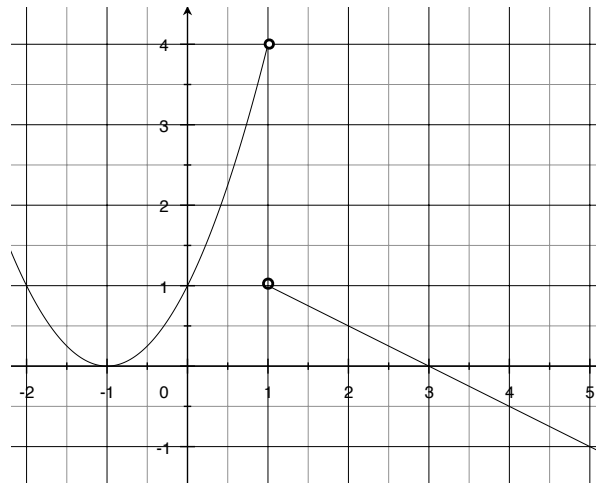
3. Graphs of f , f' , and f'' are shown above.

- (a) Estimate the x -coordinates of any local maximum points of f . Your answer(s) should be within 1 unit of the actual values. ("Round to the nearest tick mark.")

- (b) Estimate the x -coordinates of any local minimum points of f . Your answer(s) should be within 1 unit of the actual values.

- (c) Estimate the x -coordinates of any inflection points of f . Your answer(s) should be within 1 unit of the actual values.

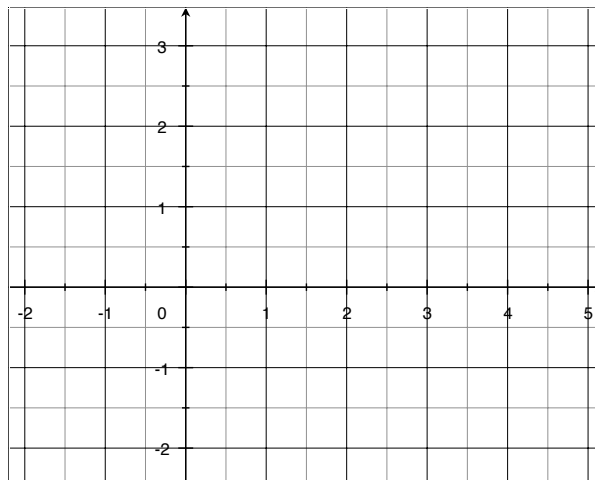
4. The function $f(x)$ is continuous for all real numbers. A graph of $f'(x)$ is shown below.



(a) On what interval(s) is f increasing? Decreasing? At what value(s) of x does f have a local maximum? Local minimum?

(b) On what interval(s) is f concave up? Concave down? At what value(s) of x does f have an inflection point?

(c) Assuming that $f(-2) = -2$, sketch a graph of f on the set of axes below.



5. Find the local and absolute maximum and minimum values (y -values) of $f(x) = x\sqrt{1-x}$ on the interval $[0, 1]$, and state the locations (x -values) at which they occur.

6. Find the absolute maximum and minimum values (y -values) of $f(x) = x^3 - 3x^2 + 1$ on the interval $[-1, 1]$, and state the locations (x -values) at which they occur.

7. The energy expended by a bird per day, E , depends on the time spent foraging for food per day, F hours. Foraging for a shorter time requires better territory, which then requires more energy for its defense. Find the foraging time that minimizes energy expenditure if

$$E = 0.25F + \frac{1.7}{F^2}$$

8. An apple tree produces, on average, 400 kg of fruit each season. However, if more than 200 trees are planted per km², crowding reduces the yield by 1kg for each tree over 200.

(a) Express the total yield from one square kilometer as a function of the number of trees on it. Graph this function.

(b) How many trees should a farmer plant on each square kilometer to maximize yield?

9. The quantity of a drug in the bloodstream t hours after a tablet is swallowed is given, in mg, by

$$q(t) = 20(e^{-t} - e^{-2t})$$

(a) How much of the drug is in the bloodstream at time $t = 0$?

(b) When is the maximum quantity of drug in the bloodstream? What is that maximum?

(c) In the long run, what happens to the quantity?

10. Evaluate the following limits, justifying your work carefully. Interpret your answers in terms of rates of growth or decay.

(a) $\lim_{x \rightarrow \infty} \frac{\ln \sqrt{x}}{x^2}$

(b) $\lim_{x \rightarrow 0^+} \frac{\cos x}{\sqrt[4]{x^5}}$

(c) $\lim_{x \rightarrow 0^+} \sin x \ln x$

11. For each function below, find the most general form of the antiderivative:

(a) $s(t) = 2 \cos t + 4t^3 - t^{-1}$

$$(b) V(x) = \frac{\cos x + 1}{\sin^2 x}$$

$$(c) u(z) = \sqrt{z}(z - 1/z)$$

12. Evaluate the definite integrals:

$$(a) \int_0^1 y^2 - y + 1 \, dy$$

$$(b) \int_0^{\ln 4} e^x + 5 \, dx$$

$$(c) \int_{-\pi}^{\pi} 2 \sin x + \cos x \, dx$$

13. Find the area under the curve $y = 1/x^2$ from $x = 1$ to $x = 3$.

14. George steadily accelerates from a stop light and records his velocity (m/s) every three seconds, resulting in the following table:

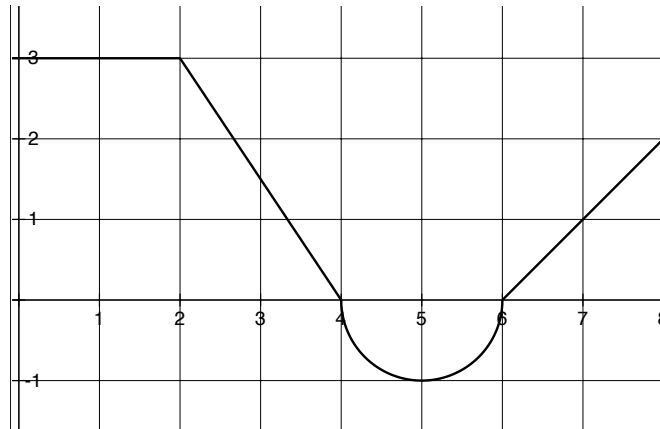
time (s)	0	3	6	9	12
velocity (m/s)	0	10	25	45	75

(a) Give lower and upper estimates for the distance George traveled in the first 12 seconds after leaving the stop light.

(b) On a sketch of velocity against time, show the lower and upper estimates of part (a).

(c) What is your best estimate for the distance George traveled in these 12 seconds?

15. Consider the function $f(x)$ below.



(a) Evaluate the integrals:

$$\int_0^4 f(x) dx$$

$$\int_4^5 f(x) dx$$

$$\int_6^8 2f(x) + 1 dx$$

(b) Let $A(x)$ be the area-so-far function with an initial x -value of $x = 4$, i.e. $A(x) = \int_4^x f(x) dx$. Evaluate $A(x)$ for the x -values given below:

x	0	2	4	6	8
$A(x)$					

(c) Let $B(x)$ be the area-so-far function with an initial x -value of $x = 0$, i.e. $B(x) = \int_0^x f(x) dx$. Evaluate $B(x)$ for the x -values given below:

x	0	2	4	6	8
$B(x)$					

(d) Let $F(x)$ be a continuous antiderivative for $f(x)$ with $F(4) = -1$. Evaluate $F(x)$ for the x -values given below:

x	0	2	4	6	8
$F(x)$					