Math 113, Additional Sample Problems to Prepare for Exam 1

To study for Exam 1, use the materials from Fall 2012 posted online (Practice Exams 1 & 2, Exams 1 & 2) as well as the homework and quizzes you have completed this semester and the sample problems below.

1. Fill in the blanks.

If $f(1) = 0$ and $f'(1) = 2$, then the tangent line to the curve $y = f(x)$ at $x = 1$ is $y = 2(x-1)$.

If $\lim_{x\to -1} f(x) = 0$ and $\lim_{x\to -1} g(x) = 1$, then $\lim_{x\to -1} \frac{f(x)}{g(x)} =$ 0.

True or false: If $\lim_{x\to -1} f(x) = 0$ and $\lim_{x\to -1} g(x) = 0$, then $\lim_{x\to -1} \frac{f(x)}{g(x)}$ does not exist. False.

↳ Example: $f(x) = x+1$, $g(x) = 2(x+1)$, $\lim_{x\to -1} f(x)/g(x) = \frac{1}{2}$

$\lim_{x\to 0} \frac{\sin x}{x} =$ 1 and $\lim_{x\to 0} \frac{\cos x - 1}{x} =$ 0

The weight, in pounds, of a child that is a years old is given by the function $W = f(a)$. The units of measurement of $f'(a)$ are lbs/year.

Meterorologists define the temperature lapse rate to be $-dT/dz$, where T is the temperature in Celcius at an altitude of z kilometers above the ground. The units of the lapse rate are °C/km.

Suppose $f(x)$ is a differentiable function. Then:

$\frac{d}{dx}\cot(x)\,f(x) =$ $-\csc^2 x\, f(x) + \cot x\, f'(x)$ $\quad \frac{d}{dx} f(e^x) =$ $e^x f'(e^x)$

$\frac{d}{dx}\frac{f(x)}{x} =$ $\frac{x f'(x) - f(x)}{x^2}$ $\quad \frac{d}{dx}(2f(x)+3) =$ $2f'(x)$

$\frac{d}{dx}\sqrt[3]{f(x)} =$ $\frac{1}{3}(f(x))^{-2/3} \cdot f'(x) = \frac{f'(x)}{3\sqrt[3]{(f(x))^2}}$

2. The thickness, P, in mm, of pelican eggshells depends on the concentration, c, of PCBs in the eggshell, measured in ppm (parts per million); that is, $P = f(c)$.

(a) The derivative $f'(c)$ is negative. What does this tell you?

The higher the concentration of PCBs, the thinner the shell.

(b) Suppose $f(200) = 0.28$. Interpret this statement in terms of PCBs and eggs. (Write a complete sentence, and make sure to include units.)

A shell that has a concentration of 200 ppm of PCBs is 0.28 mm thick.

(c) Suppose $f'(200) = -0.0005$. Interpret this statement in terms of PCBs and eggs. (Write a complete sentence, and make sure to include units.)

A shell w/ 200 ppm of PCBs will decrease in thickness by 0.0005 mm for each additional ppm of concentration of PCBs.

(d) Estimate $f(220)$, using (a) and (b). Interpret your answer in terms of PCBs and eggs. (Write a complete sentence, and make sure to include units.)

$20 \times (-0.0005) = -2 \times 0.005 = -0.01 \Rightarrow f(220) \approx 0.28 - 0.01 = 0.27$

The thickness of a shell w/ a concentration of 220 PCBs (ppm) is about 0.27 mm.

(e) Use differentials to describe your estimation in (c). In particular, make sure to state what dP and dc are.

dc = change in concentration of PCBs = 20 ppm

dP = approx. change in thickness = $f'(200) \times dc = -0.01$ mm

We estimate the actual change in thickness (ΔP) by dP to obtain our estimate for P when $c = 220$.

(f) What is the linearization of f at $a = 200$?

$$\mathcal{L}(c) = \underbrace{0.28}_{f(200)} + \underbrace{\underbrace{(-0.0005)}_{f'(200)}\underbrace{(c-200)}_{dc}}_{dP}$$