

Name: Solutions

Section: _____

You have 30 minutes to complete the following problems, without using your notes, book, or calculator.

Part 1: Algebraic and Exponential Functions

1. Classify each function given below, by circling the appropriate name or names.

(a) $f(x) = 2^x$

root power polynomial **exponential**

(b) $g(x) = x^5$

rational **power** root **polynomial**

← Note: $g(x) = x^5 = \frac{x^5}{1}$

(c) $h(x) = \frac{x}{2x-1}$

algebraic power **rational** root

so it is rational.

(d) $Q(t) = t^{1/4}$

power **root** exponential rational

(e) $P(x) = x^3 - 6x + 4$

linear power **algebraic** **polynomial**

2. State the domain of each function from the previous problem.

(a) $(-\infty, \infty)$

(b) $(-\infty, \infty)$

(c) $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

← Since, if $x = \frac{1}{2}$,
denom. would be zero.

(d) $[0, \infty)$

← Since $t^{1/4} = \sqrt[4]{t}$, and it
is impossible to get a negative
number after raising to the
4th power.

(e) $(-\infty, \infty)$

3. Find a formula for a linear function $f(x)$ whose graph passes through $(3, 1)$ and has a slope of 2.

$$y - 1 = 2(x - 3)$$

$$y = 2x - 6 + 1$$

$$y = 2x - 5$$

4. Find a formula for an exponential function $Q(t)$ whose initial value is $Q(0) = 27$ and which decreases by one third for every unit time: $Q(1) = 9$, $Q(2) = 3$, etc.

$$Q(t) = 27 \left(\frac{1}{3}\right)^t$$

Part 2: Trigonometry

5. Fill in the following table, using the five standard angles in the first quadrant.

Angle, θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
deg	rad			
0	0	0	1	0
30	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90	$\pi/2$	1	0	—

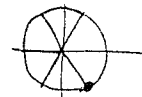
6. Evaluate the function at the given value.

(a) $\cos(-150^\circ) = \underline{\quad -\sqrt{3}/2 \quad}$

$150 = 90 + 60$



(b) $\sec(\frac{5\pi}{3}) = \underline{\quad 2 \quad}$



Part 3: Logarithms

7. Find the exact value of the given quantity:

(a) $\ln(1) = \underline{\quad 0 \quad}$

(b) $\ln(e^3) = \underline{\quad 3 \quad}$

(c) $e^{-3\ln 2} = \underline{\quad 1/8 \quad}$

$$(e^{\ln 2})^{-3} = 2^{-3} = \frac{1}{2^3}$$

8. Express $\ln 4 - \ln 5 + 3 \ln 3$ as a single logarithm: $\ln\left(\frac{108}{5}\right)$. (Show your work below.)

$$\ln\left(\frac{4}{5}\right) + \ln(27)$$

$$\ln\left(\frac{4 \cdot 27}{5}\right) = \ln\left(\frac{108}{5}\right)$$

9. Consider the function $f(x) = \ln(x+2)$. Give the domain, range, and x -intercept in the spaces provided, and sketch the graph on the axes provided. Make sure to provide scale on each axis.

Domain: $(-2, +\infty)$

$$x+2 > 0 \quad x > -2$$

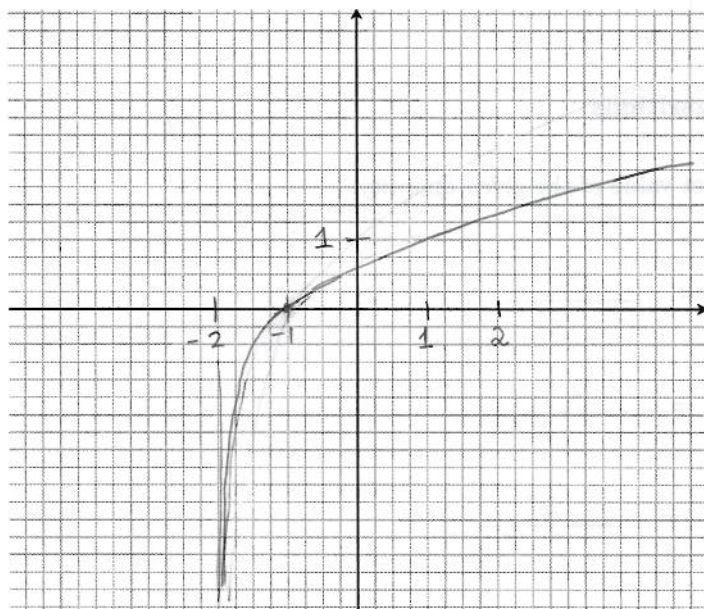
Range: \mathbb{R}

x -intercept: $x = -1$

$$0 = \ln(x+2)$$

$$1 = x+2$$

$$-1 = x$$



Part 4: Inverse Functions and Inverse Trig Functions

10. Determine whether each function is one-to-one. If it is one-to-one, sketch a graph of its inverse on the same set of axes.

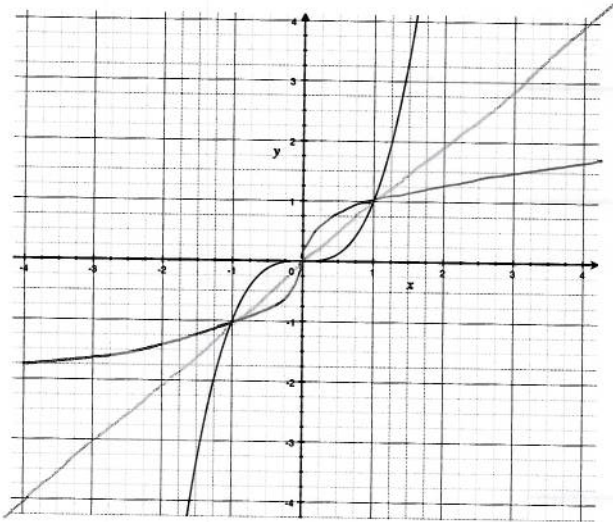


Figure 1: Is this function one-to-one? Y / N

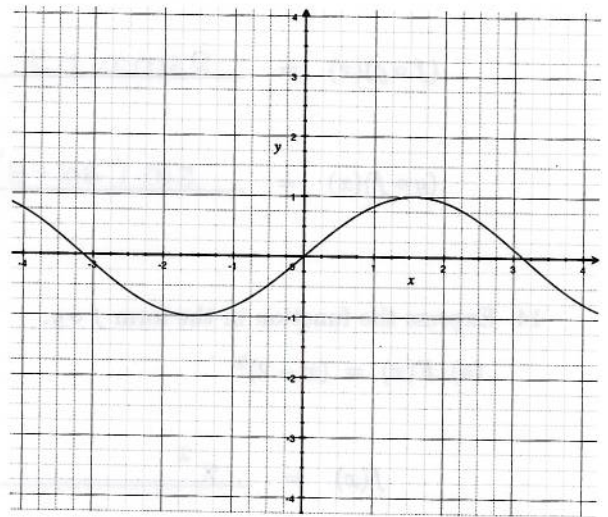


Figure 2: Is this function one-to-one? Y / N

11. Find the exact value of the given quantity:

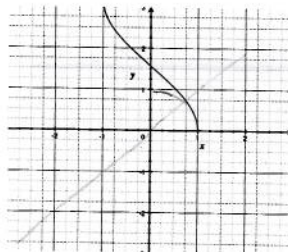
(a) $\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

(b) $\tan^{-1}(1) = \frac{\pi}{4}$

(c) $\arcsin(0) = 0$

12. The graph of which function is depicted below?

$f(x) = \arcsin(x)$ $g(x) = \cos^{-1}(x)$ $h(x) = \sec(x)$ $r(x) = \arctan(x)$



Part 5: Composition of Functions

13. Find formulas for the functions $f \circ g$ and $g \circ f$, where

$$f(x) = 2x + 3 \quad \text{and} \quad g(x) = \sin x$$

$$(f \circ g)(x) = \underline{2\sin x + 3}$$

$$(g \circ f)(x) = \underline{\sin(2x + 3)}$$

14. Express the function in the form $f \circ g$.

(a) $F(x) = (x - 2)^2$

$$f(x) = \underline{x^2}$$

$$g(x) = \underline{x - 2}$$

(b) $G(x) = \tan^2(x)$

$$f(x) = \underline{x^2}$$

$$g(x) = \underline{\tan x}$$

(c) $H(x) = \frac{2x}{2x - 3}$

$$f(x) = \underline{\frac{x}{x - 3}}$$

$$g(x) = \underline{2x}$$