Math 114, Practice Modeling with Differential Equations

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Exercises.

1. (a) Money in a bank account earns interest at a continuous annual rate of 5% times the current balance. Write a differential equation for the balance, B, in the bank account as a function of time, t, in years.

   (b) Radioactive substances decay at a rate proportional to the quantity present. Write a differential equation for the quantity, Q, of a radioactive substance present at time t. Is the constant of proportionality positive or negative?

   (c) A pollutant spilled on the ground decays at a rate of 8% a day. In addition, clean-up crews remove the pollutant at a rate of 30 gallons a day. Write a differential equation for the amount of pollutant, P, in gallons, left after t days.

   (d) Toxins in pesticides can get into the food chain and accumulate in the body. A person consumes 10 micrograms a day of a toxin, ingested throughout the day. The toxin leaves the body at a continuous rate of 3% every day. Write a differential equation for the amount of toxin, A in micrograms, in the person's body as a function of the number of days, t.

   (e) An early model of the growth of the Wikipedia assumed that every day a constant number, B, of articles are added by dedicated wikipedians and that other articles are created by the general public at a rate proportional to the number of articles already there. Express this model as a differential equation for $N(t)$, the total number of Wikipedia articles t days after January 1, 2001.

2. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate.

   (a) Let $C(t)$ be the amount of carbon dioxide in the room at after t minutes. What is $C(0)$?

   (b) Write a differential equation for the amount of carbon dioxide, C, in m^3, in the room after t minutes. (Ask yourself how much carbon dioxide is entering the room each minute, and how much is leaving the room each minute.)

   (c) Use separation of variables to obtain a formula for the general solution to this differential equation.

   (d) Given your answer in (a), find a formula for the amount of carbon dioxide in the room after t minutes.

   (e) Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run?