Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ **Section:** ⎯⎯⎯⎯⎯⎯⎯

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Bring your laptops to class! We will be using *Mathematica*!

Main Points:

1. areas inside/outside of curves, between curves, etc.

2. Use *Mathematica* to plot

1. Area inside a polar curve

When using polar coordinates, it is important to notice that the "boxes" in "polar graph paper" are not actually rectangular, and that they get larger and larger the farther away from the origin they are.

In Cartesian coordinates, our infinitessimal area is $dA = y\,dx$ (for a vertical rectangle) or $dA = x\,dy$ for a horizontal rectangle. We might expect our polar infinitessimal area to be $dA \overset{??}{=} r\,d\theta$. However, this is *not* the case, because in polar coordinates, our dA is not the area of a rectangle at all, it is the area of a sector!

If a sector has radius r and angle $d\theta$, its area is a fraction of a circle, and the fraction is determined by the ratio of $d\theta$ to 2π, the "angle" of a whole circle:

$$dA \;=\; (\text{area of circle}) \times \frac{d\theta}{2\pi} \;=\; (\pi r^2) \times \frac{d\theta}{2\pi} \;=\; \tfrac{1}{2}\,r^2 d\theta$$

Thus the area between a polar curve $r = r(\theta)$ and the origin, bounded by the rays $\theta = \theta_1$ and $\theta = \theta_2$ is:

$$A \;=\; \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2\,d\theta$$

To find the area between two polar curves, find the area inside the outer curve and the area inside the inner curve separately, and then subtract.

In general, it can be difficult to set up the integral for the area enclosed by polar curves or between polar curves, because polar curves can wrap around the origin more than once, and because there are a number of subtleties involved in finding the intersection points of polar curves.

Exercises.

1. Find the area of the region bounded by $r = e^{-\theta/4}$ in the sector $\pi/2 \le \theta \le \pi$. (The graph is provided below. No need to use *Mathematica*.)
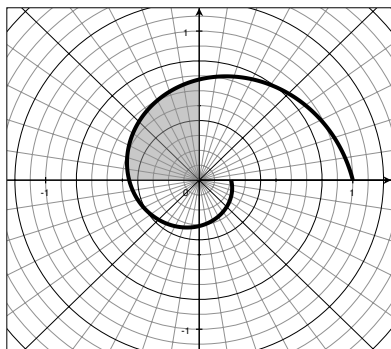
2. Using *Mathematica* to help set up the integrals

To plot a curve in polar coordinates using *Mathematica* use the `PolarPlot` command. For example to plot $r = 2\cos\theta$ from $\theta = 0$ to $\theta = 8\pi$,

    `PolarPlot[2*Cos[theta], {theta, 0, 8*Pi}]`

Remember to use capital letters for all pre-defined constants (like π) and functions (like cosine).

2. Sketch the curve $r^2 = \sin(2\theta)$ and find the area it encloses. You may use *Mathematica* to help you sketch the curve. See Example 1. (Hint: Use symmetry!)

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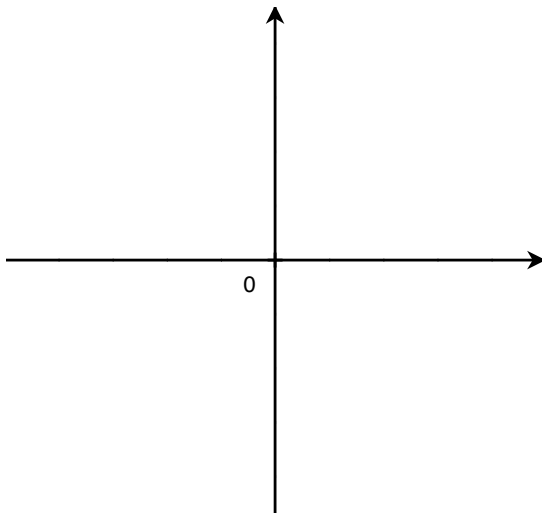
3. Find the area of the region inside $r = 1 - \sin\theta$ but outside $r = 1$. You may use *Mathematica* to help you sketch the curves. (See Example 2.)

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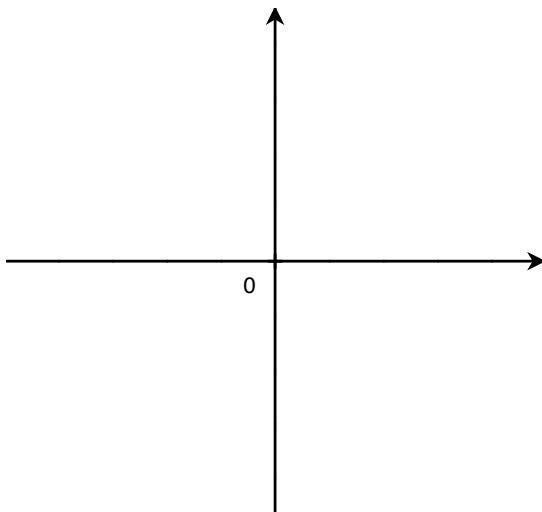
4. Find the area inside both $r = \sin 2\theta$ and $r = \sin \theta$. You may use *Mathematica* to help you sketch the curves. (Hint: divide the region into two regions.)

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